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UNIT 1: Absolute Value and Radicals (CH. 1)

1) **Arrange from least to Greatest**

   a) -|4-7|, |-(4-7)|, -|5-(-3)|, -|4|-|-7|

   -3, 3, -8, -11

   3 4 2 1

   $\boxed{-|4|-|-7|,\ -|5-(-3)|,\ -|4-7|,\ |-(4-7)|}$

2) Evaluate each expression without a calculator

(a) $\sqrt[3]{\frac{27}{8}} = \boxed{3/2}$

(b) $36^{3/2} = \sqrt[2]{36^3} = 6^3 = \boxed{216}$

(c) $\sqrt[4]{(x-4)^4} = \boxed{|x-4|}$

↑absolute value!
(this is because we can only have a positive square root)

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3) Simplify each radical expression

a) $-3\sqrt{48x^2} + 7\sqrt{75x^2}$

$-3(4x)\sqrt{3} + 7(5x)\sqrt{3}$

$= -12x\sqrt{3} + 35x\sqrt{3} = \boxed{23x\sqrt{3}}$

b) $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \boxed{= \frac{\sqrt{3}}{3}}$

c) $\frac{1}{\sqrt{14}-2} \times \frac{(\sqrt{14}+2)}{(\sqrt{14}+2)}$ → multiply by difference of squares

$= \frac{\sqrt{14}-2}{14-4} \quad \boxed{= \frac{\sqrt{14}-2}{10}}$

d) $\sqrt{x^2+4x+4} - \sqrt{x^2+12x-36}$

$\sqrt{(x+2)^2} - \sqrt{(x-6)^2}$

$= |x+2| - |x-6| = x-2-x+6 = 8$

$\boxed{= 8}$

e) $\frac{2}{3}\sqrt[3]{54x} + \frac{1}{4}\sqrt[3]{128x} \rightarrow \frac{2}{3}\sqrt[3]{27 \times 2x} + \frac{1}{4}\sqrt[3]{64 \times 2x}$

$\frac{2}{3}(3)\sqrt[3]{2x} + \frac{1}{4}(4)\sqrt[3]{2x}$

$= 2\sqrt[3]{2x} + 1\sqrt[3]{2x}$

$\boxed{= 3\sqrt[3]{2x}} \quad x \geq 0$

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4) Find the Product and Simplify:

3

(a) $(\sqrt{x+2}+3)^2$

$(\sqrt{x+2}+3)(\sqrt{x+2}+3)$

$= x+2+6\sqrt{x+2}+9$

$\boxed{= x+6\sqrt{x+2}+11}$ $\boxed{x \geq -2}$

← otherwise the solution is NOT real

(b) $\left(\frac{4-\sqrt{32}}{4}\right)^2$

$\left(\frac{4-4\sqrt{2}}{4}\right)^2 = (1-\sqrt{2})^2 = (1-\sqrt{2})(1-\sqrt{2})$

$= 1-2\sqrt{2}+2$

$= \boxed{3-2\sqrt{2}}$

5) Determine the restrictions, solve, and check solutions for extraneous roots.

Restrictions

a. $\left(\sqrt{10-3x}\right)^2 = \left(\sqrt{2x+20}\right)^2$

$10-3x \geq 0$ $2x+20 \geq 0$

$-3x \geq -10$ $2x \geq -20$

$x \leq 10/3$ $x \geq -10$

$10-3x = 2x+20$

$-10 = 5x$ $\boxed{x=-2}$ ✓ okay as it meets restrictions (not extraneous)

b. $(-\sqrt{x+2})^2 = (2)^2$

$x+2 = 4$ $x+2 \geq 0$

$x = 2$ → extraneous $x \geq -2$

because $-\sqrt{4} \neq 2$ → no solution

c. $(\sqrt{x+9})^2 = (\sqrt{1-x})^2$

$x \geq -9, x \leq -1$

$x+9 = 1-x$

$2x = -8$

$\boxed{x = -4 \rightarrow ✓}$ not extraneous

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UNIT 2: Rational Expressions (CH. 2)

1) Simplify the below rational expressions and state restrictions

a. $\frac{2x^2+x-6}{x^2+4x-5} \cdot \frac{x^3-3x^2+2x}{4x^2-6x}$

$= \frac{\cancel{(2x-3)}(x+2)}{(x+5)\cancel{(x-1)}} \cdot \frac{\cancel{x}(x-2)\cancel{(x-1)}}{2\cancel{x}\cancel{(2x-3)}}$

$\boxed{= \frac{(x+2)(x-2)}{2(x+5)} \quad x \neq -5}$

b. $\frac{x^2-14x+49}{x^2-49} \div \frac{3x-21}{x+7}$

$\frac{(x-7)(x-7)}{(x+7)(x-7)} \times \frac{(x+7)}{3(x-7)} = \frac{\cancel{(x-7)}\cancel{(x-7)}\cancel{(x+7)}}{3\cancel{(x+7)}\cancel{(x-7)}\cancel{(x-7)}} = \boxed{\frac{1}{3}}$

c. $\frac{\left(\frac{x^2-1}{x}\right)}{\frac{(x-1)^2}{x}}$

$\frac{\cancel{(x-1)}(x+1)}{\cancel{x}} \times \frac{\cancel{x}}{\cancel{(x-1)}(x-1)} = \boxed{\frac{x+1}{x-1} \quad x \neq 1}$

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2) Simplify and solve for x in the following equations.

a) $\frac{1}{x-2}+\frac{3}{x+3}=\frac{4}{x^2+x-6}$

$(x+3)(x-2)$

Common denominator is $(x+3)(x-2)$

$\boxed{x \neq 2, -3}$

$\frac{1(x+3)\cancel{(x-2)}}{\cancel{x-2}} + \frac{3(x-2)\cancel{(x+3)}}{\cancel{x+3}} = \frac{4\cancel{(x+3)(x-2)}}{\cancel{(x+3)(x-2)}}$

$x+3+3(x-2)=4$

$x+3+3x-6=4$

$4x-3=4$

$4x=7$

$\boxed{x=7/4}$ ✓

b) $\frac{6}{x+2}-\frac{3}{x^2+x-2}=\frac{x}{x^2+3x+2}$

$(x+2)(x-1)$ $(x+2)(x+1)$

Common denominator

$(x+2)(x+1)(x-1)$

$\boxed{x \neq -2, -1, 1}$

$6(x+1)(x-1) - 3(x+1) = x(x-1)$

$6x^2-6-3x-3=x^2-x$

$6x^2-9-3x=x^2-x$

$5x^2-2x-9=0$

$\boxed{x=-1.16, 1.56}$

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3. Solve the following word problems:

a) **DISTANCE Problem:** Ed is a runner and he runs a 8 km loop every day. The first 4 km, he runs at 12km/hr. He runs much slower on the way home. If it takes him 1 hour in total to run the loop, how fast is he running for the last 4 km?

D
R t

	D	R	t
There	4km	12km/hr	$\frac{4}{12}$
Back	4km	x	$\frac{4}{x}$

$$\frac{4}{12} + \frac{4}{x} = 1$$

$$4x + 48 = 12x$$

$$48 = 8x$$

$$x = 6$$

Ed runs
6 km/hr
For the last 4 km

b) **WORK Problem:** It takes Louise 2 hours to paint a room and it takes Pete 8 hours to paint the same room. How long does it take them if they paint the room together?

$$\frac{1}{2} + \frac{1}{8} = \frac{1}{x}$$

$$4x + x = 8$$

$$5x = 8$$

$$x = 8/5 \text{ hrs}$$

It takes 8/5 (1.6) hours
to paint together

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c) The sum of a number and it's reciprocal is $\frac{10}{3}$, what is the number?

$$\left[x + \frac{1}{x} = \frac{10}{3}\right] \times 3x$$

$$3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0$$

$$(3x-1)(x-3) = 0$$

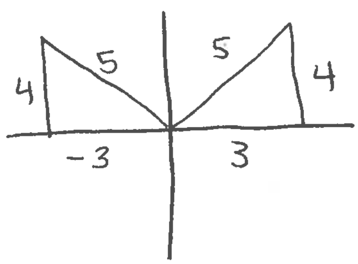
$$\boxed{x = \tfrac{1}{3} \text{ and } 3}$$

UNIT 3: Trigonometry (CH 3)

1. Given the following trigonometric ratios, draw a triangle, find the missing side and use this to solve for the two missing trigonometric ratios (sin θ, cos θ or tan θ). HINT: there should be two answers for each. (3 marks each)

S	A
T	C

a) $\sin\theta = \frac{4}{5}$



$\cos\theta = \pm\frac{3}{5}$

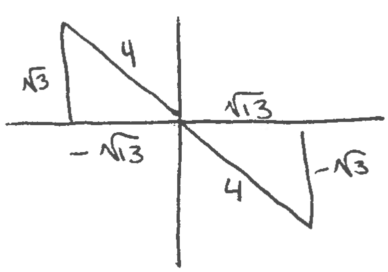
$\tan\theta = \pm\frac{4}{3}$

Answer:

$\cos\theta = \pm\frac{3}{5}$

$\tan\theta = \pm\frac{4}{3}$

b) $\tan\theta = -\frac{\sqrt{3}}{\sqrt{13}}$



Answer:

$\cos\theta = \pm\frac{\sqrt{13}}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{4}$

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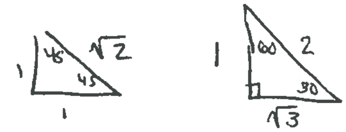
S | A
T | C



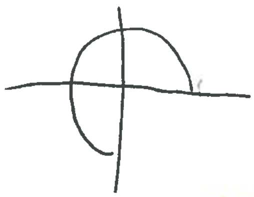
2. Evaluate exactly (without a calculator) (1 mark each)

$\sin\theta = \frac{y}{r}$

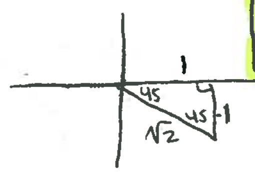
$\cos\theta = \frac{x}{r}$

$\tan\theta = y/x$

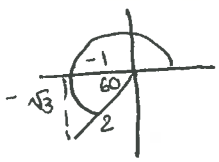
a) $\sin 270^o =$ $\boxed{-1}$



b) $\cos(-45^o) = \boxed{\frac{1}{\sqrt{2}}}$



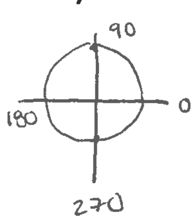
c) tan 240°= $\boxed{\sqrt{3}}$



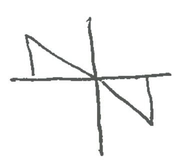
d) $\frac{\sin 135^o}{\cos(-225^o)}$ $= \boxed{-1}$

3. Find all θ for $0^o \le \theta \le 360^o$ that satisfy the given equation. There could be more than one answer! Use the unit circle and/or special triangles! (1.5 marks each)

a) $\cos\theta = 0$ $\boxed{\theta = 90°, 270°}$

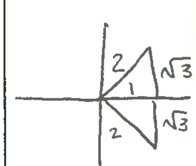


b) $\tan\theta = -1$ $\boxed{\theta = 135°, 315°}$



c) $\sin\theta = \frac{\sqrt{3}}{2}$ $\boxed{\theta = 60°, 120°}$

d) $\cos\theta = 0.5$ $\boxed{\theta = 60°, 300°}$



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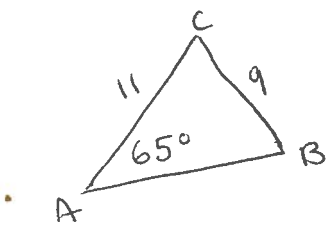
4. Use the sine law to solve the following triangles. In the case where there is no triangle, write "no solution." In the case where there is two triangles, solve for both. (2 marks each)

a) $\angle A = 65^o, a = 9, b = 11$



ASS

$$\frac{\sin 65}{9} = \frac{\sin B}{11}$$

$\sin B = 1.11$

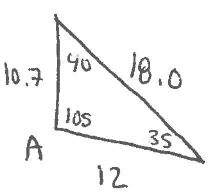
NO TRIANGLE

Answer:
NO TRIANGLE

b) $\angle A = 105^o, \angle B = 35^o, c = 12cm$

$\angle C = 40°$

AAS



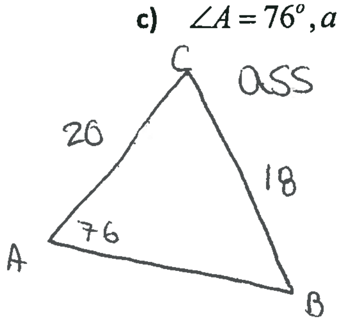
$$\frac{\sin 40}{12} = \frac{\sin 35}{b}$$

$b = 10.7$

$$\frac{\sin 40}{12} = \frac{\sin 105}{a}$$

Answer:
$b = 10.7$
$\angle C = 40°$
$a = 18.0$

c) $\angle A = 76^o, a = 25, b = 20$ *change maybe?



ass

$$\frac{\sin 76}{25} = \frac{\sin B}{20}$$

$\angle B = 50.1°$

OR ~~129.9~~ NO

$\angle B = 50.1°$

$$\frac{\sin 53.9}{c} = \frac{\sin 76}{25} \rightarrow c = 20.8$$

Answer:
$\angle B = 50.1°$
$\angle C = 53.9°$
$c = 20.8$

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5. Determine whether or not you need to use the Sine Law or the Cosine Law to solve the triangle. Then, solve the triangle (watch for no solution and 2 solutions!) (3marks each)

a) a=8, b=10, c=15

SSS

$8^2 = 10^2 + 15^2 - 2(10)(15)\cos A$

$8^2 - 10^2 - 15^2 = -300\cos A$

$-261 = -300\cos A$

$0.87 = \cos A$

$\angle A = 29.5°$

$\frac{\sin 29.5}{8} = \frac{\sin B}{10} \rightarrow \angle B = 38.0°$

Answer:
$\angle A = 29.5°$
$\angle B = 38.0°$
$\angle C = 112.5°$

b) $\angle A = 25°, a = 9, b = 20$

ASS

$\frac{\sin 25}{9} = \frac{\sin B}{20} \Rightarrow \angle B = 69.9°$ or $110.1°$ ambiguous case!

Δ1
if $\angle B = 69.9°$
$\angle C = 85.1°$
$c = 21.2$

Δ2
if $\angle B = 110.1°$
$\angle C = 44.9°$
$c = 15.0$

Δ1 $\frac{\sin 85.1}{c} = \frac{\sin 25}{9}$

OR

Δ2 $\frac{\sin 44.9°}{c} = \frac{\sin 25}{9}$

Ambiguous Case

Answer:
Δ1:
$\angle B = 69.9°$
$\angle C = 85.1°$
$c = 21.2$

Δ2
$\angle B = 110.1°$
$\angle C = 44.9°$
$c = 15.0$

c) $a = 14, b = 12, \angle C = 35°$

SSA

$c^2 = 14^2 + 12^2 - 2(12)(14)\cos 35$

$c = 8.05$

$\angle B \rightarrow \frac{\sin B}{12} = \frac{\sin 35}{8.0}$

$\angle B = 59.4°$

$\angle A = 85.6°$

Answer:
$\angle A = 85.6°$
$\angle B = 59.4°$
$c = 8.05$

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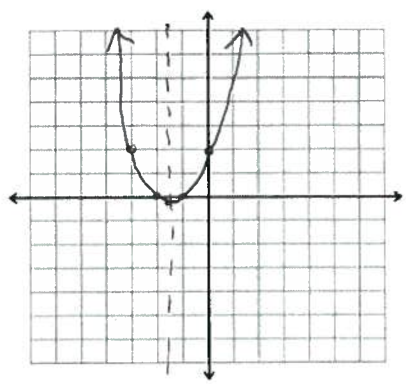
UNIT 4: Factoring and Quadratic Functions (Ch.5)

1. **Sketch the graph of the quadratic function. Identify the vertex, axis of symmetry, and x-intercept(s), domain, range and state the maximum or minimum value.**

(a) $f(x) = x^2 + 3x + 2$



$f(x) = (x+2)(x+1)$

x-int @ $x=-2$, $x=-1$

$(-2,0)\ (-1,0)$

y-int @ $y=2 \rightarrow (0,2)$

Vertex @ $(-1.5, -0.25)$

$(-\frac{3}{2}, -\frac{1}{4})$

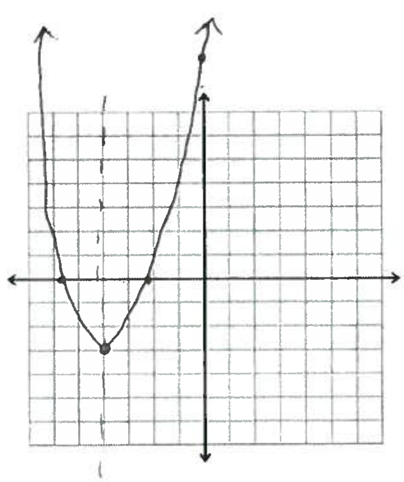
Domain: $x \in \mathbb{R}$

Range: $y \geq -\frac{1}{4}$

Min value: $y = -\frac{1}{4}$

Axis of symmetry: $x = -\frac{3}{2}$

(b) $f(x) = (x+4)^2 - 3$



Vertex: $(-4, -3)$

y-int: $y = 13$ $(0,13)$

x-int: $(-2.27, 0)\ (-5.73, 0)$

Axis of symmetry $x = -4$

Min value: $y = -3$

Domain: $x \in \mathbb{R}$

Range: $y \geq -3$

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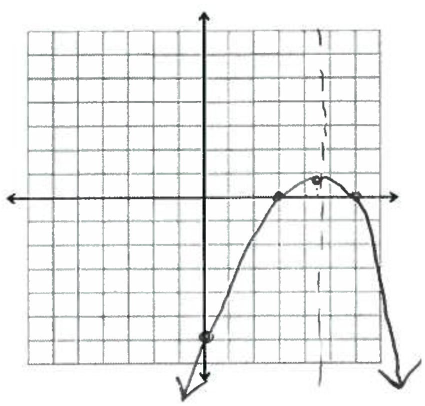
(c) $m(x) = -\frac{1}{3}x^2 + 3x - 6$



y-int: $(0, -6)$

x-int: $(3, 0)$ & $(6, 0)$

Vertex $(9/2, 3/4)$

Max Value: $y = 3/4$

Axis of Symmetry: $x = 9/2$

Domain: $x \in \mathbb{R}$

Range: $y \leq 3/4$

2. Write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.

a) Vertex: (-2, 5); Point: (0, 9)

$f(x) = a(x+2)^2 + 5$

$9 = a(0+2)^2 + 5$

$9 = 4a + 5$

$4 = 4a$

$a = 1$

$\boxed{f(x) = 1(x+2)^2 + 5}$

b) Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$; x-intercept = -2 $\rightarrow (-2, 0)$

$f(x) = a(x - 5/2)^2 - 3/4$ $\rightarrow$ Plug in $(-2, 0)$ $\quad 0 = a(-2 - \frac{5}{2})^2 - 3/4$

$\boxed{f(x) = 1/27\,(x - 5/2)^2 - 3/4}$ $\quad 3/4 = 20.25a \rightarrow a = 1/27$

3. Write the standard form of the quadratic function that passes through the following 3 points. (0,2),(6,2)(-8,4)

axis of symmetry $= \frac{6-0}{2} = 3$

vertex $= (3, k)$

$\boxed{f(x) = \frac{1}{56}(x-3)^2 + \frac{103}{56}}$

$f(x) = a(x-3)^2 + k$

$2 = a(-3)^2 + k$

$k = 2 - 9a$ $\quad K = 103/56$

$f(x) = a(x-3)^2 + 2 - 9a$

$4 = a(-8-3)^2 + 2 - 9a$

$4 = 112a + 2$ $\quad a = 1/56$

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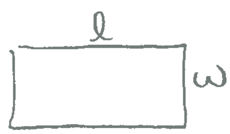
4. What is the maximum area of a rectangle that can be constructed with a perimeter of 64 cm?



$2l + 2w = 64$

$l = \frac{64 - 2w}{2} \rightarrow l = 32 - w$

Area $\rightarrow A(l) = w(32 - w)$

$A(l) = -w^2 + 32w$

$A(l) = -1(w^2 - 32w + 256) + 256$

$= -1(w - 16)^2 + 256$

max area occurs when $w = 16$

$\boxed{\text{The maximum are is } 256\text{ cm}^2}$

UNIT 5: Solving Quadratic Equations (CH. 6)

1) Solve the following Quadratic Equations (use the method of your choice)

a. $(x + 13)^2 = 25$

$x + 13 = \pm 5$

$x = 5 - 13 = -8$

$x = -5 - 13 = -18$

$\boxed{x = -8, -18}$

b. $(2x + 3)^2 - 27 = 0$

$(2x + 3)^2 = 27$

$2x + 3 = \pm\sqrt{27}$

$2x + 3 = \pm 3\sqrt{3}$

$2x = -3 \pm 3\sqrt{3}$

x

c. $(x - 7)^2 = (x + 3)^2$

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d. $\frac{1}{8}x^2 - x - 16 = 0$

$\frac{1}{8}(x^2 - 8x + \_\_\_) - 16 = 0$

$\frac{1}{8}(x^2 - 8x + 16) - 16 - 2 = 0$

$\frac{1}{8}(x-4)^2 - 18 = 0$

$(x-4)^2 = 144$

$\sqrt{(x-4)^2} = \pm\sqrt{144}$

$x - 4 = \pm 12$

$x = 4 \pm 12$

$\boxed{x = 16, -8}$

e. $3x^2 + 24x + 16 = 0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-24 \pm \sqrt{24^2 - 4(3)(16)}}{2(3)} = \frac{-24 \pm \sqrt{384}}{6} = \frac{-24 \pm 8\sqrt{}}{6}$

$\boxed{x = -0.73, -7.3}$ ← OR → $\boxed{= \frac{-12 \pm 4\sqrt{6}}{3}}$

f. $\frac{1}{4}x^2 - 2x + 7 = 0$

Solve by graphing: NO SOLUTION

g. $12x - 9x^2 = -3$

$-9x^2 + 12x + 3 = 0$

$-3(3x^2 + 4x + 1) = 0$

$-3(3x+1)(x+1) = 0$

$\boxed{x = -1/3, -1}$

h. $25x^2 + 80x + 61 = 0$

$x = \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)} = \frac{-80 \pm \sqrt{300}}{50} = \frac{-80 \pm 10\sqrt{3}}{50}$

$\boxed{x = \frac{-8 \pm \sqrt{3}}{5} = -1.25, -1.94}$

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i. $3x + 4 = 2x^2 - 7$

$2x^2 - 3x - 11 = 0$

* Solved by graphing

$\boxed{x = -1.71,\ 3.21}$

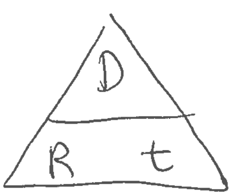
a. $2x^2 - 3x = 4x + 12$

$2x^2 - 7x - 12 = 0$

$\boxed{x = 4.76,\ -1.26}$

2. Brian decides to start training for swimming in a river. The current in the river is 4km/hr. If he swims upstream 2 km and then back downstream to where he started in 3 hours, what is his swimming speed?

x = Brian's swimming speed



	R	D	t
Up	$x-4$	2km	$\frac{2}{x-4}$
Down	$x+4$	2km	$\frac{2}{x+4}$

total time $= 3 = \text{time}_{up} + \text{time}_{down}$

$$\frac{2}{x+4} + \frac{2}{x-4} = 3$$

$$2(x-4) + 2(x+4) = 3(x-4)(x+4)$$

$$2x - 8 + 2x + 8 = 3(x^2 - 16)$$

$$4x = 3x^2 - 48$$

$$3x^2 - 4x - 48 = 0$$

$x = 4.7$ km/hr

Brian's swimming speed is 4.7 km/hr

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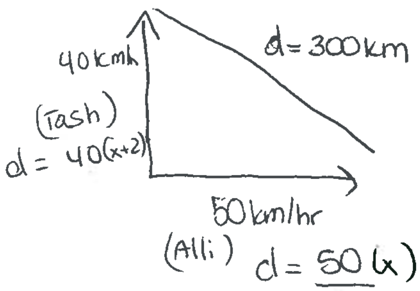
3. Natasha leaves school at 3pm and she drives north at 40 km/hr. 2 hours later (at 5pm), Alli leaves and she drives East at 50 km/hr. How long does it take before the two cars are 300 km apart?



$t_{alli} = x$

$t_{tasha} = x + 2$

d / r t

Pythagoras theorem

$(40(x+2))^2 + (50x)^2 = 300^2$

$(40x+80)^2 + 2500x^2 = 90000$

$1600x^2 + 6400x + 6400 + 2500x^2 = 90000$

$4100x^2 + 6400x - 83600 = 0$

$41x^2 + 64x - 836 = 0$

$x = 3.80$ hrs

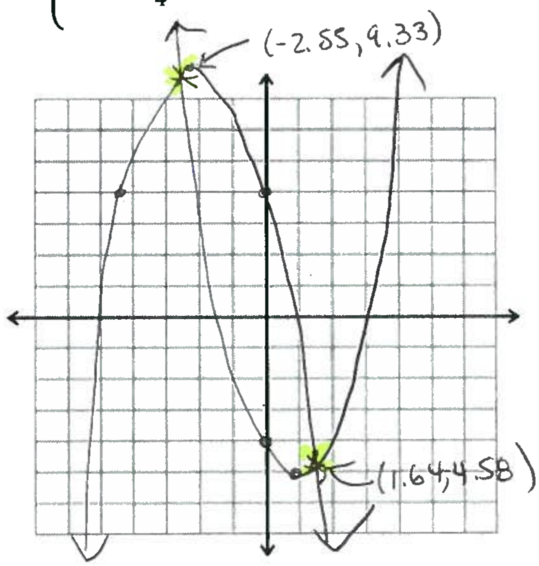
total time = tashas time = 5.8 hrs

It takes 5.8 hrs for the cars to be 300 km apart

Unit 6: SYSTEMS OF EQUATIONS (CH. 7)

1. Solve the system of equations and inequalities by graphing. If doing on calculator, sketch an accurate graph.

a) $\begin{cases} y = x^2 - 2x - 4 \\ y = -\frac{3}{4}x^2 - 4x + 4 \end{cases}$



(-2.55, 9.32)

(1.64, -4.58)

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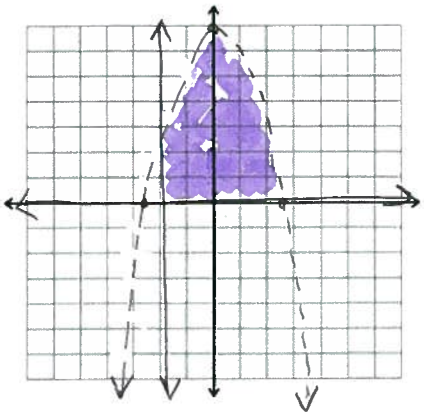
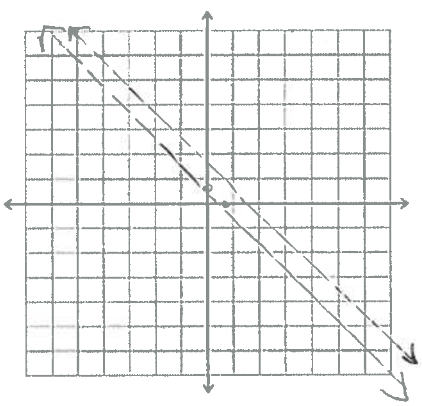
b) $\begin{cases} x^2 + y \leq 7 \\ x \geq -2 \\ y \geq 0 \end{cases}$



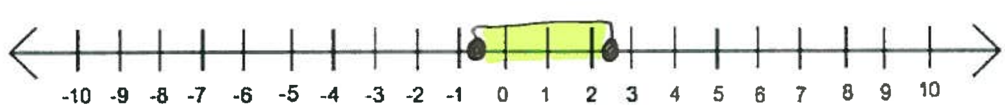
c) $\begin{cases} 2x + y > 2 \\ 6x + 3y < 2 \end{cases}$ $y < \frac{2}{3} - \frac{6x}{3}$



NO SOLUTION

d) $-2x^2 + 3x + 4 \geq 0$

$x = \frac{-3 \pm \sqrt{9 - 4(4)(-2)}}{2(-2)} = \frac{-3 \pm \sqrt{41}}{-4} = -0.85$ or 2.35



Check x=0

$4 \geq 0$ yes

* NOTICE FILLED IN CIRCLE BEC

$\boxed{-0.85 \leq x \leq 2.35}$

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2. Solve the following systems algebraically:

a) $\begin{cases} y = -x^2 - 4x + 5 \\ y = -3x + 7 \end{cases}$

$$-x^2 - 4x + 5 = -3x + 7$$

$$x^2 + x + 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(2)}}{2(1)} \leftarrow \text{NO SOLUTION}$$

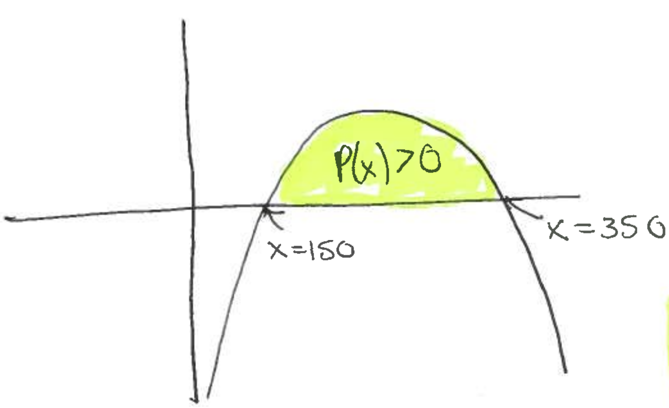
b) $\begin{cases} -7x + 6y = -4 \\ 14x - 12y = 8 \end{cases}$ $\times 2$

$$\left.\begin{aligned} -14x + 12y &= -8 \\ 14x - 12y &= 8 \end{aligned}\right\} \text{Infinite \# of solutions (same line)}$$

a) The profit for a construction company is $P(x) = -0.1x^2 + 50x - 5250$, where x is the total number of hours worked by the employees in a week. What total hours worked by the employees will produce a profit for the company?

Graphing calculator $P(x) > 0$

$$-0.1x^2 + 50x - 5250 > 0$$



$$\boxed{150 < x < 350}$$

The construction company makes a profit when the employees work between 150 and 350 hours

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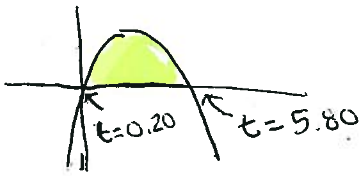
b) The height in metres of a ball thrown upward from a building is h(t)=-4.9t^2+29.4t+24.3, where "t" is the time in seconds after releasing the ball. During what time interval will the ball be above 30 meters? Graphing calculator



$30 < -4.9t^2 + 29.4t + 24.3$

$0 < -4.9t^2 + 29.4t - 5.7 \rightarrow 0.2 < t < 5.8$

the ball will be above 30 meters between 0.2 and 5.8 seconds

UNIT 7: Absolute Value, Rational and Reciprocal Functions (CH. 4)

1. Write the absolute value function as a piecewise function:

a) $g(x) = -4|x + 2| + 3$

$$g(x) = \begin{cases} -4(x+2)+3, & x \geq -2 \\ +4(x+2)+3, & x < -2 \end{cases}$$

$$\boxed{g(x) = \begin{cases} -4x-5, & x \geq -2 \\ 4x+11, & x < -2 \end{cases}}$$

b) $f(x) = \frac{1}{3}|2x - 7| + 9$

$2x-7 \geq 0$, $2x \geq 7$, $x \geq 7/2$; $2x-7<0$, $x < 7/2$

$$f(x) = \begin{cases} \frac{1}{3}(2x-7)+9, & x \geq 7/2 \\ -\frac{1}{3}(2x-7)+9, & x < 7/2 \end{cases}$$

$$f(x) = \begin{cases} \frac{2x}{3} - 7/3 + 27/3, & x \geq 7/2 \\ -2x/3 + 7/3 + \frac{27}{3}, & x < 7/2 \end{cases} \Rightarrow \boxed{f(x) = \begin{cases} \frac{2x}{3} + \frac{20}{3}, & x \geq 7/2 \\ -\frac{2x}{3} + \frac{34}{3}, & x < 7/2 \end{cases}}$$

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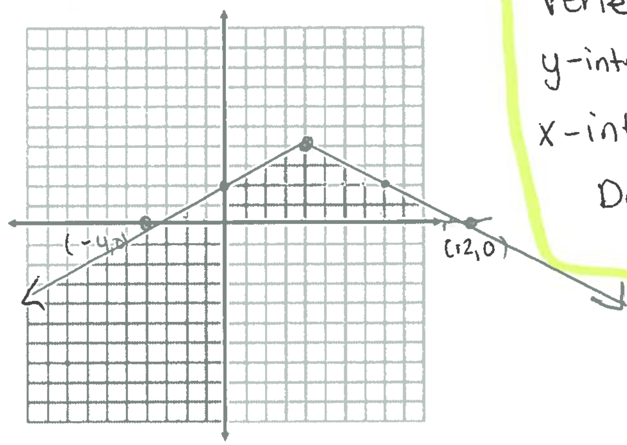
2. Graph the absolute value function. State the vertex, intercepts (x and y), domain and range.

a) $h(x) = -\frac{1}{2}|x-4|+4$



Vertex $(4,4)$
y-intercept $(0,2)$
x-intercept $(-4,0)\ (12,0)$
Domain: $x \in \mathbb{R}$
Range: $y \leq 4$

3. Solve the following absolute value functions:

a) $|x+3|=-3x$

① $x+3=-3x$

$3=-4x$

$\boxed{x=\frac{3}{-4}}$ ✓

yes

$2.25=2.25$

② $x+3=3x$

$3=2x$

$\boxed{x=\frac{3}{2}}$

X No → extraneous

$|3/2+3| \neq -3(3/2)$

$4.5 \neq -4.5$

ONLY $\boxed{x=-3/4}$

b) $|x-3|=|2x+4|$

case ① → $x-3=2x+4$

$\boxed{-7=x}$

check: $|-10|=|-10|$ ✓ yes

case ② → $-x+3=2x+4$

$-1=3x$

$\boxed{x=-1/3}$

check $|10/3|=|\frac{10}{3}|$ → yes

case ③ → $x-3=-2x-4$

$\boxed{3x=-1}$

$\boxed{x=-1/3}$ → yes!

$\boxed{x=-7,\ -1/3}$

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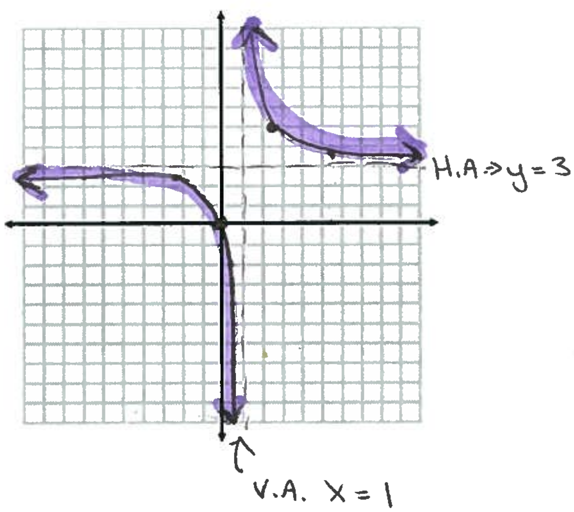
4. Sketch the following graphs and label both the horizontal and vertical asymptotes. State the domain of each function.

a) $f(x) = \frac{3x}{x-1}$

V.A. $\Rightarrow x = 1$

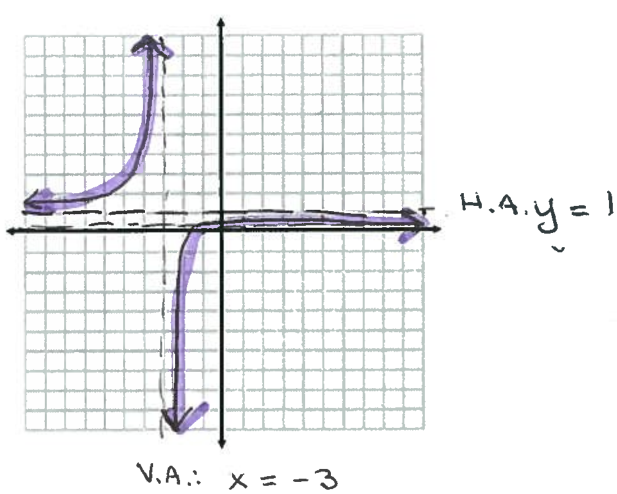
H.A $\Rightarrow y = 3$



b) $h(x) = \frac{x^2-1}{x^2+2x-3} = \frac{\cancel{(x-1)}(x+1)}{(x+3)\cancel{(x-1)}} = \frac{x+1}{x+3}$

H.A. → $y = 1$

V.A → $x = -3$



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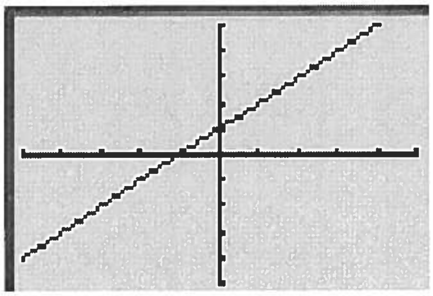
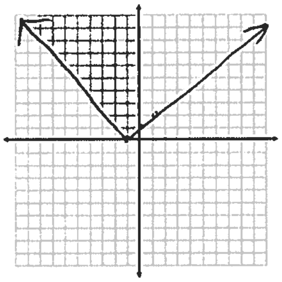
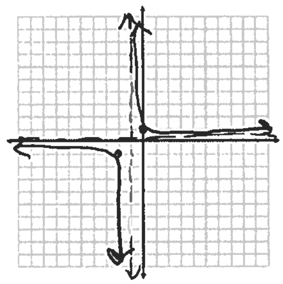
5. Given the following graph of y= f(x), sketch the graph of y = $\frac{1}{f(x)}$ and the graph of y= |f(x)|

a)



y= |f(x)

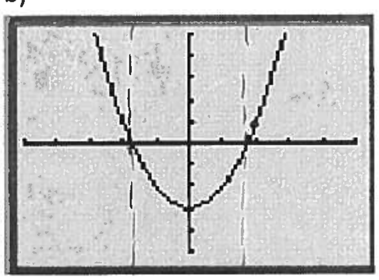


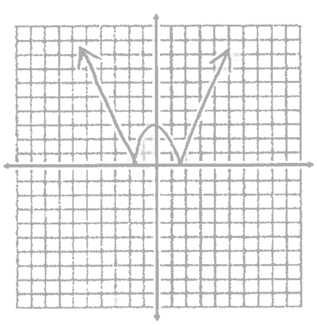


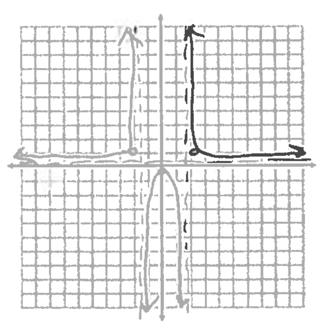
H.A. y = 0

V.A: x = -1

b)







H.A. y = 0

y= |f(x)|

V.A: x ≈ -1.8
x ≈ 1.8 } approx

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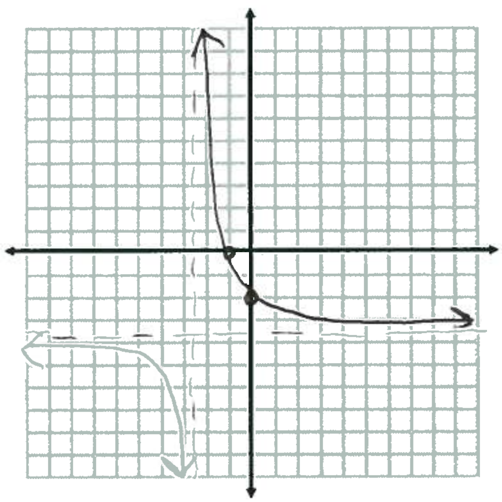
6. Sketch a graph of a rational function that has the following characteristics.

a)

x-intercept	-1
y-intercept	-2
Vertical Asymptote	x=-2.5
Horizontal Asymptote	y=-3.5



UNIT 8: Sequences and Series (CH. 8)

1. Given the series defined by $\sum_{k=2}^{15} 16\left(\frac{1}{2}\right)^{k-1}$, determine the common ratio, the number of terms and the sum.

$\underset{(k=2)}{a_1} = 16\left(\frac{1}{2}\right)^1$

$a_1 = 8$

$\underset{(k=3)}{a_2} = 16\left(\frac{1}{2}\right)^{3-1}$

$= 16\left(\frac{1}{4}\right) = 4$

$r = \frac{a_2}{a_1} = \frac{4}{8} = \frac{1}{2}$

number of terms $= 15 - 2 + 1 = 14$

$S_n = \frac{a(1-r^n)}{1-r} = \frac{8\left(1-\left(\frac{1}{2}\right)^{14}\right)}{1-\frac{1}{2}} =$

$\boxed{S_{14} = 15.999}$

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2. Find the 14th term of the sequence {3,11,19....}

$d = 8$
$a = 1$
$n = 14$

$t_{14} = a + (n-1)d$
$t_{14} = 1 + (14-1)8$
$t_{14} = 1 + 13 \times 8$

$\boxed{t_{14} = 105}$

3. Three consecutive terms of a geometric sequence are 2.5, y+3 and 9.6. Find the value of y.

$\frac{y+3}{2.5} = \frac{9.6}{y+3}$

$(y+3)(y+3) = (9.6)(2.5)$

$y^2 + 6y + 9 = (9.6)(2.5)$

$y^2 + 6y + 9 = 24$

$y^2 + 6y - 15 = 0$ → solved by graphing / reject -ve ans

$\boxed{y = 1.9 \text{ OR } -7.9}$

4. Compute the sum of the first 8 terms in the sequence {1,-3, 9.....}

↑ change*

$r = \frac{-3}{1} = -3$
$n = 8$
$a = 1$

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{1(1-(-3)^8)}{1-(-3)}$

$\boxed{S_8 = -1640}$

5. How can you tell whether or not an infinite geometric series has a finite or an infinite sum?

$|r| < 1 \rightarrow \boxed{-1 < r < 1}$ and $\boxed{r \neq 0}$ } The common difference must be between -1 and 1 and cannot = 0

6. Find the 18th term in an arithmetic sequence who's 2nd term is 11 and who's 8th term is 41.

$t_2 = 11 = a + (2-1)d$
$t_8 = 41 = a + (8-1)d$

$\rightarrow 11 = a + d$
$- (41 = a + 7d)$
$-30 = -6d$

$d = 5$
$a = 6$ } plug in to find t_{18}

$t_{18} = 6 + (18-1)5$
$t_{18} = 6 + 17(5)$
$\boxed{t_{18} = 91}$

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7. Find the 9th term in a geometric sequence who's first term is 6 and who's 4th term is -3/4.

$t_1 = a = 6$

$t_4 = ar^{n-1}$

$t_4 = 6(r)^{4-1} \rightarrow * \; t_4 = -3/4$

$-3/4 = 6(r)^3$

$-\frac{3}{24} = r^3$. $\quad -\frac{1}{8} = r^3 \rightarrow \boxed{r = \frac{-1}{2}}$

$t_9 = 6\left(\frac{-1}{2}\right)^{9-1}$

$\boxed{\therefore t_9 = \frac{3}{128}}$

8. Calculate the sum of the infinite geometric series given by

$$\sum_{k=2}^{\infty} 8\left(-\frac{1}{2}\right)^{k-1}$$

$\underset{(k=2)}{a} = 8\left(\frac{-1}{2}\right)^{2-1}$

$= 8\left(-\frac{1}{2}\right) = -4$

$\underset{(k=3)}{a_2} = 8\left(\frac{-1}{2}\right)^{3-1} = 8\left(\frac{1}{4}\right) = 2$

$r = \frac{2}{-4} = \frac{-1}{2}$

$S_\infty = \frac{a}{1-r} = \frac{-4}{1-(-1/2)} = -1.6$

$$\boxed{S_\infty = -\frac{8}{5}}$$

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Formulas

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ → quadratic formula

$$x = -\frac{b}{2a}$$

$$y = c - \frac{b^2}{4a}$$

} Vertex Formula
Vertex = (x, y)
$= \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$ } sine Law

$$a^2 = b^2 + c^2 - 2bc\cos A$$ } cosine law

$$t_n = a + (n-1)d$$ } n^{th} term of arithmetic sequence/series

$$S_n = \frac{n}{2}(2a + (n-1)d)$$ } sum of arithmetic sequence/series

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$$t_n = ar^{n-1}$$ } n^{th} term of geometric sequence/series

* Finite $$S_n = \frac{a(1-r^n)}{1-r}$$ } sum of geometric (finite) sequence/series

* infinite $$S_n = \frac{a}{1-r}$$ } sum of infinite geometric sequence/series