Name: ____________________ Date: __________

Master 1.4 Answers to Line Masters 1.1 and 1.3

Master 1.1

1. a) 6^{10} **b)** 5^6 **c)** 8^5
d) 3^{24} **e)** $3(4)^{10}$ **f)** $\frac{1}{6^5}$
g) a^4 **h)** $18a^{10}$ **i)** $\frac{9a^6}{2}$

2. a) 144 **b)** 20 **c)** –40
d) $\frac{81}{16}$ **e)** $-\frac{81}{16}$ **f)** 1
g) 6 **h)** $\frac{1}{9}$ **i)** 64

Unit 1 Exam Practice Answers.

Master 1.3 Chapter Test [redacted]

1. B **2.** B **3. a)** 8 **b)** 5

4. a) 8^2 **b)** $(-3)^3$ **c)** $36^{\frac{1}{2}}$ **d)** $(-64)^{\frac{1}{3}}$

5. rational number

6. a) rational number **b)** natural number, whole number, integer, rational number
c) rational number **d)** irrational number

7. a) i) $5\sqrt{10}$ **ii)** $2\sqrt[3]{-7}$, or $-2\sqrt[3]{7}$ **b) i)** $\sqrt{96}$ **ii)** $\sqrt[3]{135}$

8. a) 4 **b)** –32 **c)** $\frac{125}{64}$ **d)** 16

9. a) 904.8 cm^3 **b)** 4.6 cm **c)** 113.1 cm^3

10. a) i) $\frac{1}{4}$ **ii)** 150 **b) i)** 729 **ii)** 226

8. Tristan and Julie are preparing a math display for the school open house. Both students create posted to debate the following questions:
Does 0.999 … = 1 ?

Julie's Poster

$0.999\ldots \neq 1$
$0.999\ldots = 0.999\,999\,999\,999\,9\ldots$

The decimal will continue to infinity and will never reach exactly one.

Tristan's Poster

$0.999\ldots = 1$
Rewrite 0.999… in expanded form.
$$\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}+\cdots$$
This can be written as a geometric series where $t_1 = \frac{9}{10}$ and $r =$ [redacted]

a) Finish Tristan's poster by determining the value of the common ratio and then finding the sum of the infinite geometric series.

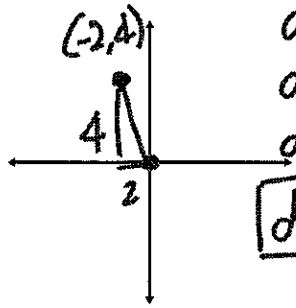
$r = \dfrac{\frac{9}{100}}{\frac{9}{10}} = \frac{9}{100} \times \frac{10}{9} = \frac{1}{10}$ $\quad S_\infty = \dfrac{t_1}{1-r} = \dfrac{\frac{9}{10}}{1-\frac{1}{10}} = \dfrac{\frac{9}{10}}{\frac{9}{10}} = \frac{9}{10} \times \frac{10}{9} = 1$

b) Which student do you think correctly answered the question? Explain.

Tristan! At infinity it will reach 1

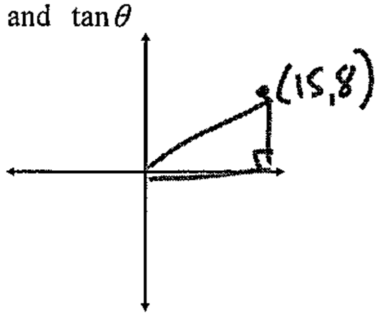
Chapter ~~2~~ 5: Trigonometry

1. Determine the exact distance, in simplified form, from the origin to a point P (-2, 4) on the terminal arm of an angle.



$d^2 = 2^2 + 4^2$
$d^2 = 20$
$d = \sqrt{20}$
$\boxed{d = 2\sqrt{5}}$

2. Point P (15,8) is on the terminal arm of angle θ. Determine the exact values for $\sin\theta$, $\cos\theta$ and $\tan\theta$



$d^2 = 8^2 + 15^2$
$d^2 = 289$
$d = 17$

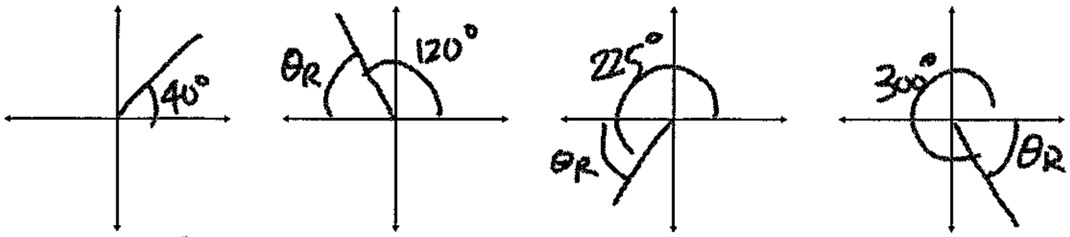
$\sin\theta = \frac{8}{17}$

$\cos\theta = \frac{15}{17}$

$\tan\theta = \frac{8}{15}$

3. Sketch each angle in standard position and determine the measure of the reference angle.

a) 40° b) 120° c) 225° d) 300°



$\theta_R = 40°$ $\theta_R = 60°$ $\theta_R = 45°$ $\theta_R = 60°$

4. Determine the **exact** value of each trigonometric ratio.

a) $\sin 405°$ + $\theta_R = 45°$

$\sin 45° = \boxed{\frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}}$

b) $\cos 330°$ + $\theta_R = 30°$

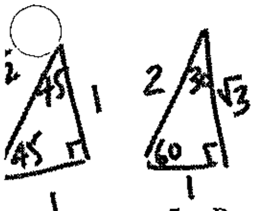
$\cos 30° = \boxed{\frac{\sqrt{3}}{2}}$

c) $\tan 225°$ + $\theta_R = 45°$

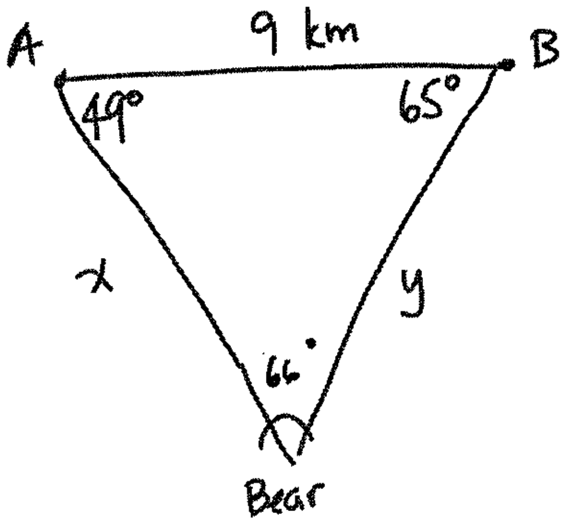
$\tan 45° = \boxed{1}$

d) $\cos 150°$ − $\theta_R = 30°$

$\cos 30° = \frac{\sqrt{3}}{2} \rightarrow \boxed{-\frac{\sqrt{3}}{2}}$



5. Radio collars are used to track polar bears by sending signals via GPS to receiving stations. Two receiving stations are 9 km apart along a straight road. At station A, the signal from one of the collars comes from a direction of 49° from the road. At station B, the signal from the same collar comes from a direction of 65° from the road. Determine the distance the polar bear is from each of the stations.



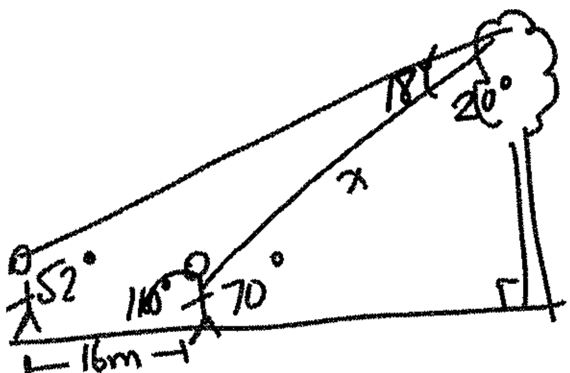
$\angle Bear = 66°$

$\frac{x}{\sin 65°} = \frac{9}{\sin 66°}$ $\boxed{x = 8.9 \text{ km}}$

$\frac{y}{\sin 49°} = \frac{9}{\sin 66°}$ $\boxed{y = 7.4 \text{ km}}$

6. Waterton Lakes National Park in Alberta is a popular site for birdwatching, with over 250 species of birds recorded. Chelsea spots a rare pileated woodpecker in a tree at an angle of elevation of 52°. After walking 16 m closer to the tree she determines the new angle of elevation to be 70°.

a) Sketch and label a diagram to represent the situation.

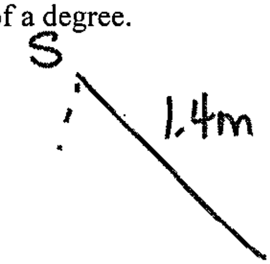


$$\frac{16}{\sin 18^\circ} = \frac{x}{\sin 52^\circ}$$

$$\boxed{x = 40.8\text{ m}}$$

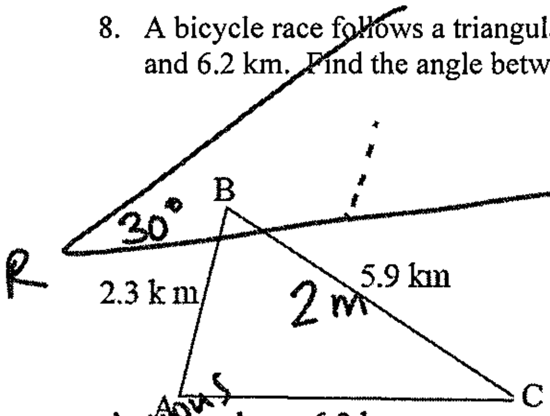
b) What is the closest distance that Chelsea is from the bird, to the nearest tenth of a meter?

7. In ΔRST, RT = 2 m, ST = 1.4 m, and $\angle R = 30°$. Determine the measure of obtuse $\angle S$ to the nearest tenth of a degree.



$$\frac{\sin 30^\circ}{14} = \frac{\sin S}{2}$$

8. A bicycle race follows a triangular course. The 3 legs of the race are in order 2.3 km, 5.9 km and 6.2 km. Find the angle between the starting leg and the finishing leg.



Ambiguous case!

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$5.9^2 = 6.2^2 + 2.3^2 - 2(6.2)(2.3)(\cos A)$$
$$34.81 = 38.44 + 5.29 - 28.52\cos A$$
$$34.81 = 43.73 - 28.52\cos A$$
$$34.81 - 43.73 = -28.52\cos A$$
$$\frac{34.81 - 43.73}{-28.52} = \cos A$$
$$< A = \cos^{-1}\left(\frac{-8.92}{-28.52}\right)$$
$$< A \cong 72^o$$

$\sin S = 0.714$

$\angle S = 45.36$ (acute)

obtuse $\boxed{\angle S = 134.4^\circ}$

∴ The angle between the starting leg and the finishing leg is 72°.

Ch 4 answers here too.

Chapter 3: Quadratic Functions

1. Match each characteristic with the correct function.

Characteristic		*Quadratic Function*
I) vertex in quadrant III	C	A $y=-5(x-2)^2-3$
II) opens downward	A	B $y=3(x+3)^2+5$
III) axis of symmetry: x = 3	D	C $y=2(x+2)^2-3$
IV) range: $\{y\mid y\geq 5, y\varepsilon R\}$	B	D $y=3(x-3)^2-5$

2. Classify each as a quadratic function or a function that is not quadratic.

a) $y=(x+6)-1$ Not Quadratic

b) $y=-5(x+1)^2$ Quadratic

c) $y=\sqrt{(x+2)^2}+7$ Not Quadratic

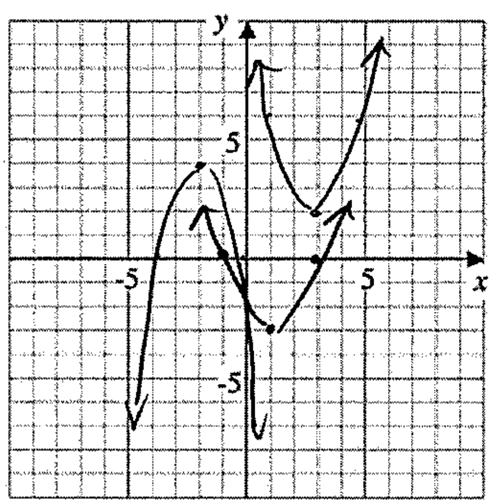
d) $y+8=x^2$ Quadratic

3. Sketch a possible graph for a quadratic function given each set of characteristics.

a) axis of symmetry: x = -2, range: $\{y\mid y\leq 4, y\varepsilon R\}$

b) axis of symmetry: x = 3, range: $\{y\mid y\geq 2, y\varepsilon R\}$

c) opens upward, vertex at (1, -3), one x-intercept at the point (3,0).



4. Identify the vertex, domain, range, axis of symmetry, x-intercepts and y-intercept for each quadratic function.

a) $f(x)=(x+4)^2-3$

V: (-4, -3)

axis: x = -4

D: x ∈ R

R: y ≥ -3

x-int = -5.73, -2.26

y-int = 13

b) $f(x)=-(x-2)^2+1$

V: (2, 1)

axis: x = 2

D: x ∈ R

R: y ≤ 1

x-int: 1, 3

y-int: -3

c) $f(x)=-2x^2-6$

V: (0, -6)

axis: x = 0

D: x ∈ R

R: y ≤ -6

x-int: N/A

y-int: -6

d) $f(x)=\frac{1}{2}(x+8)^2+6$

V: (-8, 6)

axis: x = -8

D: x ∈ R

R: y ≥ 6

x-int: N/A

y-int: 38

5. Rewrite each function in the form $y=a(x-p)^2+q$. Compare the graph of each function to the graph of $y=x^2$.

a) $y=x^2-10x+18$

$y = x^2 - 10x + 25 - 25 + 18$

$y = (x-5)^2 - 7$

shifted down 7
right 5

b) $y=-x^2+4x-7$

$y = -(x^2 - 4x + 4 - 4) - 7$

$y = -(x-2)^2 - 3$

shifted down 3
right 2

opens down

c) $y=3x^2-6x+5$

$y = 3(x^2 - 2x + 1 - 1) + 5$

$y = 3(x-1)^2 + 2$

shifted up 2
right 1

stretched 3

d) $y=\frac{1}{4}x^2+4x+20$

$y = \frac{1}{4}(x^2 + x + \frac{1}{4} - \frac{1}{4}) + 20$

$y = \frac{1}{4}(x + \frac{1}{2})^2 - \frac{1}{16} + 20$

$y = \frac{1}{4}(x + \frac{1}{2})^2 + \frac{319}{16}$

shifted up 319/16
left 1/2

6. **a)** The approximate height, h, in meters, of an arrow shot into the air with an initial velocity of 20 m/s after t seconds can be modeled by the function $h(t) = -5t^2 + 20t + 2$. What is the maximum height reached by the arrow?

$h(t) = -5(t^2 - 4t + 4 - 4) + 2$

$= -5(t-2)^2 + 22$

max height = 22 m

vertex $(2, 22)$

b) From what height was the arrow shot?

y-int.

$= -5(0-2)^2 + 22$

$= -5(4) + 22 = 2$

height = 2 m

c) How long did it take for the arrow to hit the ground, to the nearest second?

x-int (calc) $= 4.1$ or 4 seconds

Chapter 4: Quadratic Equations

7. Solve by the indicated method.

FACTORING

a) $x^2 - 4x = -3$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$

b) $9x^2 + 6x - 8 = 0$

$(9x+12)(9x-6) = 0$

$(3x+4)(3x-2) = 0$

$x = -\frac{4}{3}, \frac{2}{3}$

COMPLETING THE SQUARE

c) $2(x-3)^2 - 8 = 0$

$2(x-3)^2 = 8$

$(x-3)^2 = 4$

$x - 3 = \pm 2$

$x = \pm 2 + 3$

d) $-\frac{1}{2}(x+2)^2 + 1 = -4$

$-\frac{1}{2}(x+2)^2 = -5$

$(x+2)^2 = 10$

$x + 2 = \pm\sqrt{10}$

$x = \sqrt{10} - 2, -\sqrt{10} - 2$

QUADRATIC FORMULA (leave exact answers please!)

e) $3x^2 + 19x - 14 = 0$

$\frac{-19 \pm \sqrt{(19)^2 - 4(3)(-14)}}{2(3)}$

$\frac{-19 \pm \sqrt{529}}{6}$

$\frac{-19 \pm 23}{6}$ $x = \frac{2}{3}, -7$

f) $2x^2 - 4x - 3 = 0$

$\frac{4 \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$

$\frac{4 \pm \sqrt{40}}{4}$

$\frac{4 \pm 2\sqrt{10}}{4}$

$x = 1 + \frac{1}{2}\sqrt{10},\ 1 - \frac{1}{2}\sqrt{10}$

8. The sum of the squares of three consecutive integers is 194. What are the integers?

$x^2 + (x+1)^2 + (x+2)^2 = 194$

$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 194$

$3x^2 + 6x + 5 = 194$

$3x^2 + 6x - 189 = 0$

$x^2 + 2x - 63 = 0$

$(x+9)(x-7) = 0$

$x = -9, 7$

$-9, -8, -7$

or

$7, 8, 9$

9. The Empress Theatre, in Fort Macleod, is Alberta's oldest continually operating theatre. Much of the theatre is the same as when it was constructed in 1912, including the 285 original seats on the main floor. The number of rows on the main floor is 4 more than the number of seats in each row. Determine the number of rows and the number of seats in each row.

Let x = # of seats/row

then $x + 4$ = # of rows.

$x(x+4) = 285$

$x^2 + 4x - 285 = 0$

$(x+19)(x-15) = 0$

$x = -19, 15$

There are 15 seats/row and 19 rows.

10. Use the discriminant to determine the nature of the roots for each quadratic equation. $b^2 - 4ac$

a) $x^2 - 6x + 3 = 0$

$(-6)^2 - 4(1)(3)$

$= 24$

2 real, distinct roots

b) $x^2 + 22x + 121 = 0$

$(22)^2 - 4(1)(121)$

$= 0$

2 real, equal roots

c) $-x^2 + 3x = 5$

$-x^2 + 3x - 5 = 0$

$(3)^2 - 4(-1)(-5)$

$= -11$

no real roots

Chapter ~~5~~ 6: Radical Expressions & Equations

1. Express $3xy\sqrt[3]{2x}$ as an entire radical.

$= \sqrt[3]{2x \cdot 3^3 \cdot x^3 \cdot y^3}$

$= \sqrt[3]{54x^4y^3}$

2. Express $\sqrt{48a^3b^2c^5}$ as a simplified mixed radical.

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c \cdot c}$

$= 4abc^2\sqrt{3ac}$

3. Order the set of numbers from least to greatest.

$3\sqrt{6}, \sqrt{36}, 2\sqrt{3}, \sqrt{18}, 2\sqrt{9}, \sqrt[3]{8}$

$= \sqrt{54}, \sqrt{36}, \sqrt{12}, \sqrt{18}, \sqrt{36}, \sqrt[3]{8}$

$\sqrt[3]{8} < \sqrt{12} < \sqrt{18} < \sqrt{36} \leq \sqrt{36} < \sqrt{54}$

4. Simplify each expression. Identify any restrictions on the values for the variables.

a) $4\sqrt{2a} + 5\sqrt{2a}$

$= 9\sqrt{2a}$

$a \geq 0$

b) $10\sqrt{20x^2} - 3x\sqrt{45}$

$= 10\sqrt{2 \cdot 2 \cdot 5 \cdot x \cdot x} - 3x\sqrt{5 \cdot 3 \cdot 3}$

$= 20x\sqrt{5} - 9x\sqrt{5}$

$= 11x\sqrt{5}$

no restrictions

5. Simplify. Identify any restrictions on the values of the variable in part c).

a) $2\sqrt[3]{4}\left(-4\sqrt[3]{6}\right)$

$= -8\sqrt[3]{24}$

$= -8\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$

$= \boxed{-16\sqrt[3]{3}}$

b) $\sqrt{6}\left(\sqrt{12} - \sqrt{3}\right)$

$= \sqrt{72} - \sqrt{18}$

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} - \sqrt{2 \cdot 3 \cdot 3}$

$= 6\sqrt{2} - 3\sqrt{2} = \boxed{3\sqrt{2}}$

c) $\left(6\sqrt{a} + \sqrt{3}\right)\left(2\sqrt{a} - \sqrt{4}\right)$ FOIL

$= 12\sqrt{a^2} - 6\sqrt{4a} + 2\sqrt{3a} - \sqrt{12}$

$a \geq 0$

$= \boxed{12a - 12\sqrt{a} + 2\sqrt{3a} - 2\sqrt{3}}$

6. Rationalize each denominator.

a) $\frac{\sqrt{12}}{\sqrt{4}} \times \frac{\sqrt{4}}{\sqrt{4}}$

$= \frac{\sqrt{48}}{4} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}}{4}$

$= \frac{\cancel{4}\sqrt{3}}{\cancel{4}} = \boxed{\sqrt{3}}$

b) $\frac{2}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

$= \quad 4 - 2\sqrt{3}$

c) $\frac{(\sqrt{7}+\sqrt{28})}{(\sqrt{7}-\sqrt{14})} \times \frac{(\sqrt{7}+\sqrt{14})}{(\sqrt{7}+\sqrt{14})}$

$= \quad -3 - 3\sqrt{2}$ 2

7. Solve the radical equation $(\sqrt{x+6})^2 = (x)^2$. Verify your anwer(s).

$x + 6 = x^2$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$x = 3, -2$

Check.

$\sqrt{3+6} = 3$

$\sqrt{9} = 3$ ✓

$\sqrt{-2+6} = -2$

$\sqrt{4} = -2$ ✗

(reject)

$\boxed{x = 3}$

8. On a children's roller coaster ride, the speed in a loop depends on the height of the hill the car has just come down and the radius of the loop. The velocity, v, in feet per second, of a car at the top of a loop of radius r, in feet, is given by the formula $v = \sqrt{h-2r}$, where h is the height of the previous hill, in feet.

a) Find the height of the hill when the velocity at the top of the loop is 20 ft/s and the radius of the loop is 15 ft.

$v = 20$

$r = 15$

$h = ?$

$20 = \sqrt{h - 2(15)}$

$(20)^2 = (\sqrt{h-30})^2$

$400 = h - 30$

$430 = h$

height = 430 ft.

b) Would you expect the velocity of the car to increase or decrease as the radius of the loop increases? Explain your reasoning.

as $r \uparrow$ $v \uparrow$ as per equation.

$v = \sqrt{h - 2r}$

Chapter 6: Rational Expressions & Equations

9. Simplify each expression. Identify any non-permissible values.

a) $\dfrac{12\cancel{a^2}\cancel{b}}{48\cancel{a^2}b^4}$ 3

npv's: $a \neq 0$, $b \neq 0$

$\boxed{\dfrac{1}{4b^3}}$

b) $\dfrac{4-x}{x^2-8x+16}$

$-\dfrac{\cancel{(4-x)}}{\cancel{(x-4)}(x-4)}$

$x \neq 4$

$= \boxed{\dfrac{-1}{x-4}}$

c) $\dfrac{(x-3)(x+5)}{x^2-1} \div \dfrac{x+2}{x-3}$ $x \neq \pm 1, 3, -2$

$\dfrac{(x-3)(x+5)}{(x+1)(x-1)} \div \dfrac{x+2}{x-3}$

$\boxed{\dfrac{(x-3)(x+5)(x-3)}{(x+1)(x-1)(x+2)}}$

d) $\dfrac{5x-10}{6x} \times \dfrac{3x}{15x-30}$ $x \neq 0, 2$

$= \dfrac{\cancel{5}\cancel{(x-2)}}{2\,\cancel{6x}} \times \dfrac{\cancel{3x}}{3\,\cancel{15}\cancel{(x-2)}}$

$= \boxed{\dfrac{1}{6}}$

f) $\left(\dfrac{x+2}{x-3}\right)\left(\dfrac{x^2-9}{x^2-4}\right) \div \left(\dfrac{x+3}{x-2}\right)$ $x \neq \pm 2, 3, -3$

$= \dfrac{\cancel{(x+2)}(x+3)\cancel{(x-3)}}{\cancel{(x-3)}\cancel{(x+2)}(x-2)} \div \dfrac{(x+3)}{(x-2)}$

$= \cancel{\dfrac{x+3}{x-2}} \times \cancel{\dfrac{x-2}{x+3}} = \boxed{1}$

10. Determine the sum or difference. Express answers in lowest terms. Identify any non-permissible values.

a) $\dfrac{10}{a+2} + \dfrac{a-1}{a-7}$ $a \neq -2, 7$

$= \dfrac{10(a-7) + (a-1)(a+2)}{(a+2)(a-7)}$

$= \dfrac{10a - 70 + a^2 + 2a - a - 2}{(a+2)(a-7)}$

$= \boxed{\dfrac{a^2 + 11a - 72}{(a+2)(a-7)}}$

b) $\dfrac{3x+2}{x+4} - \dfrac{x-5}{x^2-4}$ TYPO $x \neq 4, -4$

$= \dfrac{(3x+2)(x-4) - (x-5)(x+4)}{(x-4)(x+4)}$

$= \dfrac{3x^2 - 12x + 2x - 8 - x^2 - 4x + 5x + 20}{(x-4)(x+4)}$

$= \boxed{\dfrac{2x^2 - 9x + 12}{(x-4)(x+4)}}$

c) $\dfrac{2x}{x^2-25}-\dfrac{3}{x^2-4x-5}$ $x \neq \pm 5, -1$

$\dfrac{2x}{(x+5)(x-5)} - \dfrac{3}{(x-5)(x+1)}$

$\dfrac{2x(x+1)-3(x+5)}{LCD}$

$= \dfrac{2x^2+2x-3x-15}{LCD}$

$= \dfrac{2x^2-x-15}{LCD}$

$= \dfrac{(2x-6)(2x+5)}{LCD}$

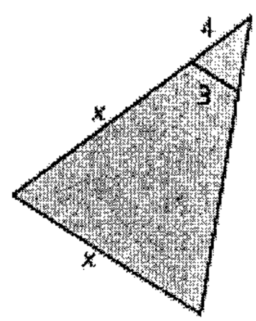
$\boxed{= \dfrac{(x-3)(2x+5)}{(x+5)(x-5)(x+1)}}$

11. Sandra simplified the expression $\dfrac{(x+2)(x+5)}{x+5}$ to $x+2$. She stated that they were equivalent expressions. Do you agree or disagree with Sandra's statement? Explain.

Not exactly. Must state $x \neq -2$

12. When two triangles are similar, you can use the proportion of corresponding sides to determine an unknown dimension. Solve the rational equation to determine the value of x.

$$\frac{x+4}{4} = \frac{x}{3}$$



$3(x+4) = x(4)$

$3x + 12 = 4x$

$\boxed{12 = x}$

Chapter 7: Absolute Value and Reciprocal Functions

13. Order the values from least to greatest.

$-8-5$

$|-5|, |4-6|, |2(-4)-5|, |8.4|$

~~5, 2, 13, 8.4~~

$2 < 5 < 8.4 < 13$

Radicals Worksheet Name: KEY

Evaluate the Perfect Squares

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2	13^2	14^2	15^2
1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

1. Simplify the following.

$\sqrt{1}$ $=1$ $\quad$ $\sqrt{4}$ $=2$ $\quad$ $\sqrt{9}$ $=3$ $\quad$ $\sqrt{16}$ $=4$ $\quad$ $\sqrt{25}$ $=5$ $\quad$ $\sqrt{36}$ $=6$

$\sqrt{144}$ $=12$

$\sqrt{\frac{4}{9}}$ $=\frac{2}{3}$

$\sqrt{\frac{9}{100}}$ $=\frac{3}{10}$

$\sqrt{\frac{9}{36}}$ $=\frac{3}{6}=\frac{1}{2}$

$\sqrt{\frac{25}{100}}$ $=\frac{5}{10}=\frac{1}{2}$

$\sqrt{\frac{900}{100}}$ $=\sqrt{9}$ $=3$

2. Find the value of the following.

$\sqrt{49}$ $=7$

$2\sqrt{16}$ $=2\cdot 4$ $=8$

$25\sqrt{4}$ $=25\cdot 2$ $=50$

$\sqrt{9}+\sqrt{36}$ $=3+6$ $=9$

$5\sqrt{4}+10\sqrt{9}$ $=5\cdot 2+10\cdot 3$ $=10+30$ $=40$

3. Convert the following entire radicals to mixed radicals in simplest form.

$\sqrt{12}$ $=\sqrt{4}\cdot\sqrt{3}$ $=2\sqrt{3}$

$\sqrt{27}$ $=\sqrt{9}\cdot\sqrt{3}$ $=3\sqrt{3}$

$\sqrt{32}$ $=\sqrt{16}\cdot\sqrt{2}$ $=4\sqrt{2}$

$\sqrt{60}$ $=\sqrt{4}\cdot\sqrt{15}$ $=2\sqrt{15}$

$\sqrt{72}$ $=\sqrt{36}\cdot\sqrt{2}$ $=6\sqrt{2}$

$\sqrt{242}$ $=\sqrt{121}\cdot\sqrt{2}$ $=11\sqrt{2}$

4. Simplify the following.

$\sqrt{200}$ $=\sqrt{100}\cdot\sqrt{2}$ $=10\sqrt{2}$

$\sqrt{36}$ $=6$

$\sqrt{45}$ $=\sqrt{9}\cdot\sqrt{5}$ $=3\sqrt{5}$

$\sqrt{49}$ $=7$

$\sqrt{64}$ $=8$

$\sqrt{108}$ $=\sqrt{36}\cdot\sqrt{3}$ $=6\sqrt{3}$

5. Simplify the following.

$\sqrt{18} = \sqrt{9}\cdot\sqrt{2}$
$= 3\sqrt{2}$

$5\sqrt{24} = 5\sqrt{4}\cdot\sqrt{6}$
$= 5\cdot 2\sqrt{6}$
$= 10\sqrt{6}$

$\sqrt{15} = \sqrt{15}$

$4\sqrt{20} = 4\cdot\sqrt{4}\cdot\sqrt{5}$
$= 4\cdot 2\sqrt{5}$
$= 8\sqrt{5}$

$6\sqrt{16} = 6\cdot 4$
$= 24$

$7\sqrt{17} = 7\sqrt{17}$

$8\sqrt{18} = 8\sqrt{9}\cdot\sqrt{2}$
$= 8\cdot 3\sqrt{2}$
$= 24\sqrt{2}$

$10\sqrt{98} = 10\sqrt{49}\cdot\sqrt{2}$
$= 10\cdot 7\cdot\sqrt{2}$
$= 70\sqrt{2}$

$\sqrt{18} = \sqrt{9}\cdot\sqrt{2}$
$= 3\sqrt{2}$

$5\sqrt{24} = 5\sqrt{4}\cdot\sqrt{6}$
$= 5\cdot 2\sqrt{6}$
$= 10\sqrt{6}$

$\sqrt{32} = \sqrt{16}\cdot\sqrt{2}$
$= 4\sqrt{2}$

$\sqrt{200} = \sqrt{100}\cdot\sqrt{2}$
$= 10\sqrt{2}$

6. Convert the following radicals to mixed radicals in simplest form.

$3\sqrt{12}$	$2\sqrt{32}$	$5\sqrt{24}$	$6\sqrt{98}$	$4\sqrt{200}$	$8\sqrt{18}$
$= 3\sqrt{4}\cdot\sqrt{3}$	$= 2\sqrt{16}\cdot\sqrt{2}$	$= 5\sqrt{4}\cdot\sqrt{6}$	$= 6\sqrt{49}\cdot\sqrt{2}$	$= 4\sqrt{100}\cdot\sqrt{2}$	$= 8\sqrt{9}\sqrt{2}$
$= 6\sqrt{3}$	$= 8\sqrt{2}$	$= 10\sqrt{6}$	$= 42\sqrt{2}$	$= 40\sqrt{2}$	$= 24\sqrt{2}$

7. Simplify the following.

$4\sqrt{8}$	$2\sqrt{16}$	$16\sqrt{18}$	$32\sqrt{9}$	$25\sqrt{25}$	$4\sqrt{1}$
$= 4\sqrt{4}\cdot\sqrt{2}$	$= 2\cdot 4$	$= 16\sqrt{9}\sqrt{2}$	$= 32\cdot 3$	$= 25\cdot 5$	$= 4$
$= 8\sqrt{2}$	$= 8$	$= 48\sqrt{2}$	$= 96$	$= 125$	

8. Simplify the following.

$4\sqrt{4}$	$2\sqrt{81}$	$9\sqrt{25}$	$3\sqrt{9}$	$100\sqrt{2\bullet 18}$	$\sqrt{16\sqrt{16}}$
$= 4\cdot 2$	$= 2\cdot 9$	$= 9\cdot 5$	$= 3\cdot 3$	$= 100\sqrt{36}$	$= \sqrt{16\cdot 4}$
$= 8$	$= 18$	$= 45$	$= 9$	$= 100\cdot 6$	$= 4\cdot 2$
				$= 600$	$= 8$

9. Convert the following mixed radicals to entire radicals.

$3\sqrt{2}$	$2\sqrt{3}$	$5\sqrt{6}$	$6\sqrt{7}$	$4\sqrt{10}$	$2\sqrt{11}$
$= \sqrt{9}\cdot\sqrt{2}$	$= \sqrt{4}\cdot\sqrt{3}$	$= \sqrt{25}\sqrt{6}$	$= \sqrt{36}\cdot\sqrt{7}$	$= \sqrt{16}\cdot\sqrt{10}$	$= \sqrt{4}\cdot\sqrt{11}$
$= \sqrt{18}$	$= \sqrt{12}$	$= \sqrt{150}$	$= \sqrt{252}$	$= \sqrt{160}$	$= \sqrt{44}$

Evaluate the Perfect Cubes

1^3	2^3	3^3	4^3	5^3	6^3	7^3	8^3	9^3	10^3
1	8	27	64	125	216	343	512	729	1000

1. Evaluate the following.

$\sqrt[3]{1}$ $= 1$

$\sqrt[3]{-8}$ $= -2$

$\sqrt[3]{27}$ $= 3$

$\sqrt[3]{\frac{27}{8}}$ $= \frac{3}{2}$

$\sqrt[3]{\frac{1000}{125}}$ $= \frac{10}{5} = 2$

$\sqrt[3]{\frac{10}{270}}$ $= \sqrt[3]{\frac{1}{27}}$ $= \frac{1}{3}$

2. Simplify the following. (mixed radical in simplest form where possible)

$\sqrt[3]{16} = \sqrt[3]{8}\cdot\sqrt[3]{2}$
$= 2\sqrt[3]{2}$

$\sqrt[3]{54} = \sqrt[3]{27}\cdot\sqrt[3]{2}$
$= 3\sqrt[3]{2}$

$\sqrt[3]{48} = \sqrt[3]{8}\cdot\sqrt[3]{6}$
$= 2\sqrt[3]{6}$

$5\sqrt[3]{2000} = 5\sqrt[3]{1000}\cdot\sqrt[3]{2}$
$= 5\cdot 10\sqrt[3]{2}$
$= 50\sqrt[3]{2}$

3. Convert the following mixed radicals to entire radicals.

$2\sqrt[3]{3} = \sqrt[3]{8}\cdot\sqrt[3]{3}$
$= \sqrt[3]{24}$

$3\sqrt[3]{2} = \sqrt[3]{27}\cdot\sqrt[3]{2}$
$= \sqrt[3]{54}$

$2\sqrt[3]{4} = \sqrt[3]{8}\cdot\sqrt[3]{4}$
$= \sqrt[3]{32}$

$5\sqrt[3]{10} = \sqrt[3]{125}\cdot\sqrt[3]{10}$
$= \sqrt[3]{1250}$

4. Simplify the following. (mixed radical in simplest form where possible)

$2\sqrt[3]{16} = 2\sqrt[3]{8}\cdot\sqrt[3]{2}$
$= 2\cdot 2\sqrt[3]{2}$
$= 4\sqrt[3]{2}$

$\sqrt[3]{216} = 6$

$4\sqrt[3]{8} = 4\cdot 2$
$= 8$

$3\sqrt[3]{8000} = 3\sqrt[3]{8}\cdot\sqrt[3]{1000}$
$= 3\cdot 2\cdot 10$
$= 60$

Radicals & Variables

1. Simplify the following.

$\sqrt{x^2}$ $= x$

$\sqrt{x^4}$ $= x^2$

$\sqrt{x^6}$ $= x^3$

$\sqrt{x^{10}y^8}$ $= x^5y^4$

$\sqrt{16x^{16}}$ $= 4x^8$

$\sqrt{36x^8}$ $= 6x^4$

2. Simplify the following. (mixed radical in simplest form where possible)

$\sqrt{x^3}$ $= \sqrt{x^2}\cdot\sqrt{x}$ $= x\sqrt{x}$

$\sqrt{x^5}$ $= \sqrt{x^4}\cdot\sqrt{x}$ $= x^2\sqrt{x}$

$\sqrt{x^7}$ $= \sqrt{x^6}\cdot\sqrt{x}$ $= x^3\sqrt{x}$

$\sqrt{x^{15}}$ $= \sqrt{x^{14}}\cdot\sqrt{x}$ $= x^7\sqrt{x}$

$\sqrt{9x^9}$ $= 3\sqrt{x^8}\sqrt{x}$ $= 3x^4\sqrt{x}$

$\sqrt{18x^7}$ $\sqrt{9}\cdot\sqrt{x^6}\cdot\sqrt{2x}$ $= 3x^3\sqrt{2x}$

$\sqrt{12x^3y^6}$ $= \sqrt{4}\sqrt{x^2}y^3\sqrt{3x}$ $= 2xy^3\sqrt{3x}$

$\sqrt{50x^{11}y^5}$ $= \sqrt{25}\sqrt{x^{10}}\sqrt{y^4}\sqrt{2xy}$ $= 5x^5y^2\sqrt{2xy}$

$\sqrt{36x^{16}y^9}$ $= 6x^8\sqrt{y^8}\cdot\sqrt{y}$ $= 6x^8y^4\sqrt{y}$

3. Simplify the following. (mixed radical in simplest form where possible)

$\sqrt[3]{8x^3} = 2x$

$\sqrt[3]{16x^{12}}$ $= \sqrt[3]{8}\,x^4\sqrt[3]{2}$ $= 2x^4\sqrt[3]{2}$

$3\sqrt[3]{16x^{10}}$ $= 3\sqrt[3]{8}\sqrt[3]{x^9}\sqrt[3]{2x}$ $= 6x^3\sqrt[3]{2x}$

$\sqrt[3]{27x^8y^4}$ $= 3\sqrt[3]{x^6}\sqrt[3]{y^3}\sqrt[3]{x^2y}$ $= 3x^2y\sqrt[3]{x^2y}$

4. The square root of large numbers

$\sqrt{720}$ $= \sqrt{144}\cdot\sqrt{5}$ $= \boxed{12\sqrt{5}}$

2)720, 2)360, 2)180, 2)90, 5)45, 3)9, 3)3, 1

$2\cdot2\cdot2\cdot2\cdot3\cdot3\cdot5 = 720$

$(2\cdot2\cdot3)(2\cdot2\cdot3)\cdot5 = 720$

$12^2\cdot5 = 720$

$\sqrt{1944}$ $= \sqrt{18^2}\cdot\sqrt{6}$ $= \boxed{18\sqrt{6}}$

2)1944, 2)972, 2)486, 3)243, 3)81, 3)27, 3)9, 3)3, 1

$2\cdot2\cdot2\cdot3\cdot3\cdot3\cdot3\cdot3 = 1944$

$(2\cdot3\cdot3)(2\cdot3\cdot3)(2\cdot3) = 1944$

$18^2\cdot6 = 1944$

$\sqrt{3375}$ $= \sqrt{15^2}\cdot\sqrt{15}$ $= \boxed{15\sqrt{15}}$

5)3375, 5)675, 5)135, 3)27, 3)9, 3)3, 1

$3\cdot3\cdot3\cdot5\cdot5\cdot5 = 3375$

$(3\cdot5)(3\cdot5)(3\cdot5) = 3375$

$15^2\cdot15 = 3375$

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M10C Quiz – Exponents & Radicals C1-C4 Name: KEY

Evaluate each of the following and record on the Numerical Response Answer Sheet provided:

1. $\sqrt{81}$ $= \boxed{9}$

2. $\sqrt[3]{64}$ $= \boxed{4}$

3. $\dfrac{\sqrt{144}}{36^{\frac{1}{2}}}$ $= \dfrac{12}{6} = \boxed{2}$

4. $\dfrac{\sqrt{100}}{\sqrt[3]{8}}$ $= \dfrac{10}{2} = \boxed{5}$

Express each entire radical as a mixed radical in simplest form. Your final answer should be in the form $a\sqrt{b}$ or $a\sqrt[3]{b}$. For each question please record the value of a followed by the value of b on the Numerical Response Answer Sheet provided.

5. $\sqrt{50}$ $= \sqrt{25}\cdot\sqrt{2}$
$= \boxed{5\sqrt{2}}$

6. $\sqrt{20}$ $= \sqrt{4}\cdot\sqrt{5}$
$= \boxed{2\sqrt{5}}$

7. $3\sqrt{27}$ $= 3\cdot\sqrt{9}\cdot\sqrt{3}$
$= \boxed{9\sqrt{3}}$

8. $\sqrt{48}$ $= \sqrt{16}\cdot\sqrt{3}$
$= \boxed{4\sqrt{3}}$

9. $\sqrt[3]{32}$ $= \sqrt[3]{8}\cdot\sqrt[3]{4}$
$= \boxed{2\sqrt[3]{4}}$

10. $2\sqrt[3]{54}$ $= 2\sqrt[3]{27}\cdot\sqrt[3]{2}$
$= \boxed{6\sqrt[3]{2}}$

Express each mixed radical as an entire radical. Your final answer should be in the form $\sqrt{b}$ or $\sqrt[3]{b}$. For each question please record the value of b on the Numerical Response Answer Sheet provided.

11. $5\sqrt{10}$ $= \sqrt{25}\cdot\sqrt{10}$
$= \boxed{\sqrt{250}}$

12. $2\sqrt[3]{5}$ $= \sqrt[3]{8}\cdot\sqrt[3]{5}$
$= \boxed{\sqrt[3]{40}}$

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UNIT 1: Absolute Value and Radicals (CH. 1)

1) Arrange from least to Greatest

a) -|4-7|, |-(4-7)|, -|5-(-3)|, -|4|-|-7|

~~-3 , 3 , -8 , -11~~

~~3 4 2 1~~

~~1 2 3 4~~

~~$\boxed{-|4|-|-7| \ , \ -|5-(-3)| \ , \ -|4-7| \ , \ |-(4-7)|}$~~

2) Evaluate each expression without a calculator

(a) $\sqrt[3]{\frac{27}{8}}$ $= \boxed{\frac{3}{2}}$

(b) $36^{3/2}$ $= \sqrt[2]{36^3} = 6^3 = \boxed{216}$

(c) $\sqrt[4]{(x-4)^4}$ $= \boxed{|x-4|}$

absolute value!
(this is because we can only have a positive square root)

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3) Simplify each radical expression

a) $-3\sqrt{48x^2}+7\sqrt{75x^2}$

$-3(4x)\sqrt{3} + 7(5x)\sqrt{3}$

$= -12x\sqrt{3} + 35x\sqrt{3} = \boxed{23x\sqrt{3}}$

b) $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \boxed{= \frac{\sqrt{3}}{3}}$

c) $\frac{1}{\sqrt{14}-2} \times \frac{(\sqrt{14}+2)}{(\sqrt{14}+2)}$ → multiply by difference of squares

$= \frac{\sqrt{14}-2}{14-4} \quad \boxed{= \frac{\sqrt{14}-2}{10}}$

d) $\sqrt{x^2+4x+4}-\sqrt{x^2+12x-36}$

$\sqrt{(x+2)^2} - \sqrt{(x-6)^2}$

$= |x+2| - |x-6| = x-2-x+6 = 8$

$\boxed{= 8}$

e) $\frac{2}{3}\sqrt[3]{54x}+\frac{1}{4}\sqrt[3]{128x} \rightarrow \frac{2}{3}\sqrt[3]{27 \times 2x} + \frac{1}{4}\sqrt[3]{64 \times 2x}$

$\frac{2}{3}(3)\sqrt[3]{2x} + \frac{1}{4}(4)\sqrt[3]{2x}$

$= 2\sqrt[3]{2x} + 1\sqrt[3]{2x}$

$\boxed{= 3\sqrt[3]{2x}} \quad x \geq 0$

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4) Find the Product and Simplify:

3

(a) $(\sqrt{x+2}+3)^2$

$(\sqrt{x+2}+3)(\sqrt{x+2}+3)$

$= x+2+6\sqrt{x+2}+9$

$\boxed{= x+6\sqrt{x+2}+11}$ $\boxed{x \geq -2}$

otherwise the solution is NOT real

(b) $(\frac{4-\sqrt{32}}{4})^2$

$\left(\frac{4-4\sqrt{2}}{4}\right)^2 = (1-\sqrt{2})^2 = (1-\sqrt{2})(1-\sqrt{2})$

$= 1-2\sqrt{2}+2$

$= \boxed{3-2\sqrt{2}}$

5) Determine the restrictions, solve, and check solutions for extraneous roots.

Restrictions

a. $(\sqrt{10-3x})^2 = (\sqrt{2x+20})^2$

$10-3x \geq 0$ $2x+20 \geq 0$

$-3x \geq -10$ $2x \geq -20$

$x \leq 10/3$ $x \geq -10$

$10-3x = 2x+20$

$-10 = 5x$ $\boxed{x=-2}$ ✓ okay as it meets restrictions (not extraneous)

b. $(-\sqrt{x+2})^2 = (2)^2$

$x+2 = 4$ $x+2 \geq 0$

$x = 2$ → extraneous because $-\sqrt{4} \neq 2$ → no solution $x \geq -2$

c. $(\sqrt{x+9})^2 = (\sqrt{1-x})^2$

$x \geq -9, x \leq -1$

$x+9 = 1-x$

$2x = -8$

$\boxed{x = -4 \rightarrow ✓}$ not extraneous

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UNIT 2: Rational Expressions (CH. 2)

1) Simplify the below rational expressions and state restrictions

a. $\frac{2x^2+x-6}{x^2+4x-5} \cdot \frac{x^3-3x^2+2x}{4x^2-6x}$

$= \frac{\cancel{(2x-3)}(x+2)}{(x+5)\cancel{(x-1)}} \cdot \frac{\cancel{x}(x-2)\cancel{(x-1)}}{2\cancel{x}\cancel{(2x-3)}}$

$\boxed{= \frac{(x+2)(x-2)}{2(x+5)} \quad x \neq -5}$

b. $\frac{x^2-14x+49}{x^2-49} \div \frac{3x-21}{x+7}$

$\frac{(x-7)(x-7)}{(x+7)(x-7)} \times \frac{(x+7)}{3(x-7)} = \frac{\cancel{(x-7)}\cancel{(x-7)}\cancel{(x+7)}}{3\cancel{(x+7)}\cancel{(x-7)}\cancel{(x-7)}} = \boxed{\frac{1}{3}}$

c. $\frac{\left(\frac{x^2-1}{x}\right)}{\frac{(x-1)^2}{x}}$

$\frac{\cancel{(x-1)}(x+1)}{\cancel{x}} \times \frac{\cancel{x}}{\cancel{(x-1)}(x-1)} = \boxed{\frac{x+1}{x-1} \quad x \neq 1}$

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2) Simplify and solve for x in the following equations.

a) $\frac{1}{x-2}+\frac{3}{x+3}=\frac{4}{x^2+x-6}$

$(x+3)(x-2)$

Common denominator is $(x+3)(x-2)$

$\boxed{x \neq 2, -3}$

$\frac{1(x+3)\cancel{(x-2)}}{\cancel{x-2}} + \frac{3(x-2)\cancel{(x+3)}}{\cancel{x+3}} = \frac{4\cancel{(x+3)}\cancel{(x-2)}}{\cancel{(x+3)}\cancel{(x-2)}}$

$x+3+3(x-2)=4$

$x+3+3x-6=4$

$4x-3=4$

$4x=7$

$\boxed{x=7/4}$ ✓

b) $\frac{6}{x+2}-\frac{3}{x^2+x-2}=\frac{x}{x^2+3x+2}$

$(x+2)(x-1)$ $(x+2)(x+1)$

Common denominator

$(x+2)(x+1)(x-1)$

$\boxed{x \neq -2, -1, 1}$

$6(x+1)(x-1) - 3(x+1) = x(x-1)$

$6x^2-6-3x-3=x^2-x$

$6x^2-9-3x=x^2-x$

$5x^2-2x-9=0$

$\boxed{x=-1.16, 1.56}$

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3. Solve the following word problems:

a) **DISTANCE Problem:** Ed is a runner and he runs a 8 km loop every day. The first 4 km, he runs at 12km/hr. He runs much slower on the way home. If it takes him 1 hour in total to run the loop, how fast is he running for the last 4 km?

6

	D	R	t
There	4km	12km/hr	$\frac{4}{12}$
Back	4km	x	$\frac{4}{x}$

$$\frac{4}{12} + \frac{4}{x} = 1$$

$$4x + 48 = 12x$$

$$48 = 8x$$

$$x = 6$$

Ed runs
6 km/hr
For the last 4 km

b) **WORK Problem:** It takes Louise 2 hours to paint a room and it takes Pete 8 hours to paint the same room. How long does it take them if they paint the room together?

$$\frac{1}{2} + \frac{1}{8} = \frac{1}{x}$$

$$4x + x = 8$$

$$5x = 8$$

$$x = 8/5 \text{ hrs}$$

It takes 8/5 (1.6) hours
to paint together

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c) The sum of a number and it's reciprocal is $\frac{10}{3}$, what is the number?

$$\left[x + \frac{1}{x} = \frac{10}{3}\right] \times 3x$$

$$3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0$$

$$(3x-1)(x-3) = 0$$

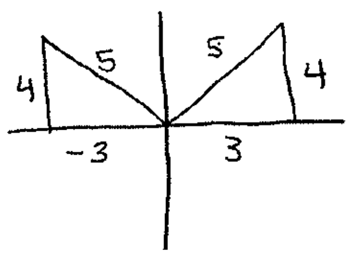
$$\boxed{x = \tfrac{1}{3} \text{ and } 3}$$

UNIT 3: Trigonometry (CH 3)

1. Given the following trigonometric ratios, draw a triangle, find the missing side and use this to solve for the two missing trigonometric ratios (sin θ, cos θ or tan θ). HINT: there should be two answersfor each. (3 marks each)

S	A
T	C

a) $\sin\theta = \frac{4}{5}$



$\cos\theta = \pm\frac{3}{5}$

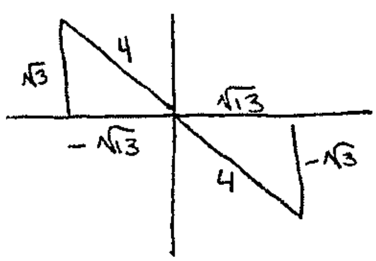
$\tan\theta = \pm\frac{4}{3}$

Answer:

$\cos\theta = \pm\frac{3}{5}$

$\tan\theta = \pm\frac{4}{3}$

b) $\tan\theta = -\frac{\sqrt{3}}{\sqrt{13}}$



Answer:

$\cos\theta = \pm\frac{\sqrt{13}}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{4}$

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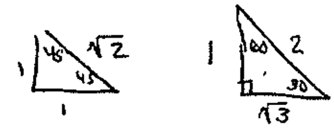
Name:__________________________________

S	A
T	C

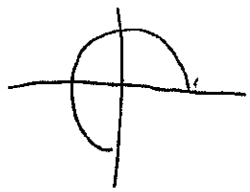
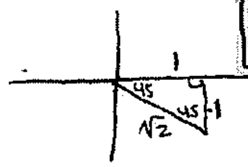
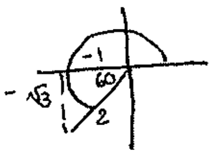


2. Evaluate exactly (without a calculator) (1 mark each)

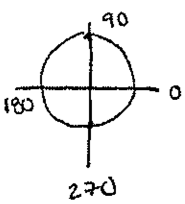
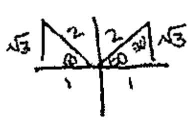
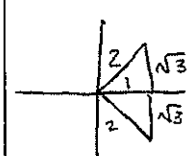
$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = y/x$

a) $\sin 270^\circ =$ $\boxed{-1}$ 	**b)** $\cos(-45^\circ) = \boxed{\frac{1}{\sqrt{2}}}$ 
c) $\tan 240^\circ =$ $\boxed{\sqrt{3}}$ 	**d)** $\dfrac{\sin 135^\circ}{\cos(-225^\circ)} = \boxed{-1}$ 

3. Find all θ for $0^\circ \le \theta \le 360^\circ$ that satisfy the given equation. There could be more than one answer! Use the unit circle and/or special triangles! (1.5 marks each)

a) $\cos\theta = 0$ $\boxed{\theta = 90^\circ, 270^\circ}$ 	**b)** $\tan\theta = -1$ $\boxed{\theta = 135^\circ, 315^\circ}$ 
c) $\sin\theta = \dfrac{\sqrt{3}}{2}$ $\boxed{\theta = 60^\circ, 120^\circ}$ 	**d)** $\cos\theta = 0.5$ $\boxed{\theta = 60^\circ, 300^\circ}$ 

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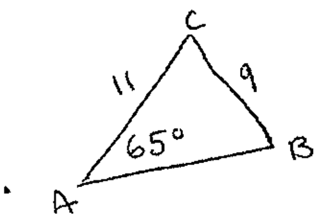
4. **Use the sine law to solve the following triangles. In the case where there is no triangle, write "no solution." In the case where there is two triangles, solve for both.** (2 marks each)

a) $\angle A = 65°, a = 9, b = 11$



ASS

$$\frac{\sin 65}{9} = \frac{\sin B}{11}$$

$\sin B = 1.11$

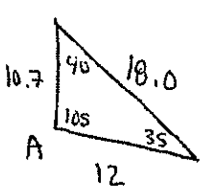
NO TRIANGLE

Answer:
NO TRIANGLE

b) $\angle A = 105°, \angle B = 35°, c = 12cm$

$\angle C = 40°$

AAS



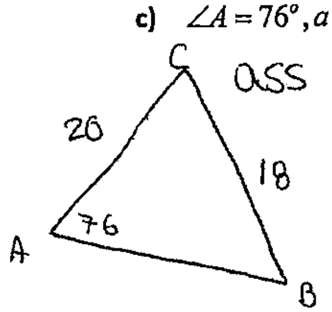
$$\frac{\sin 40}{12} = \frac{\sin 35}{b}$$

$b = 10.7$

$$\frac{\sin 40}{12} = \frac{\sin 105}{a}$$

Answer:
$b = 10.7$
$\angle C = 40°$
$a = 18.0$

c) $\angle A = 76°, a = 25, b = 20$ *change maybe?



ASS

$$\frac{\sin 76}{25} = \frac{\sin B}{20}$$

$\angle B = 50.1°$

OR $\cancel{129.9}$ NO

$\angle B = 50.1°$

$$\frac{\sin 53.9}{c} = \frac{\sin 76}{25} \rightarrow c = 20.8$$

Answer:
$\angle B = 50.1°$
$\angle C = 53.9°$
$c = 20.8$

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(10)

5. Determine whether or not you need to use the Sine Law or the Cosine Law to solve the triangle. Then, solve the triangle (watch for no solution and 2 solutions!) (3marks each)

a) a=8, b=10, c=15

SSS

$\to 8^2 = 10^2 + 15^2 - 2(10)(15)\cos A$

$8^2 - 10^2 - 15^2 = -300 \cos A$

$-261 = -300 \cos A$

$0.87 = \cos A$

$\angle A = 29.5°$

$\to \frac{\sin 29.5}{8} = \frac{\sin B}{10} \to \angle B = 38.0°$

Answer:
$\angle A = 29.5°$
$\angle B = 38.0°$
$\angle C = 112.5°$

b) $\angle A = 25°, a = 9, b = 20$

ASS

$\frac{\sin 25}{9} = \frac{\sin B}{20} \Rightarrow \angle B = 69.9°$ or $110.1°$ } ambiguous case!

Δ1
if $\angle B = 69.9°$
$\angle C = 85.1°$
$c = 21.2$

Δ2
if $\angle B = 110.1°$
$\angle C = 44.9°$
$c = 15.0$

Δ1 $\frac{\sin 85.1}{c} = \frac{\sin 25}{9}$

$\boxed{\text{OR}}$

Δ2 $\frac{\sin 44.9°}{c} = \frac{\sin 25}{9}$

Ambiguous Case

Answer:
Δ1:
$\angle B = 69.9°$
$\angle C = 85.1°$
$c = 21.2$

Δ2
$\angle B = 110.1°$
$\angle C = 44.9°$
$c = 15.0$

c) $a = 14, b = 12, \angle C = 35°$

SSA

$c^2 = 14^2 + 12^2 - 2(12)(14)\cos 35$

$c = 8.05$

$\angle B \to \frac{\sin B}{12} = \frac{\sin 35}{8.0}$

$\angle B = 59.4°$

$\angle A = 85.6°$

Answer:
$\angle A = 85.6°$
$\angle B = 59.4°$
$c = 8.05$

8/10

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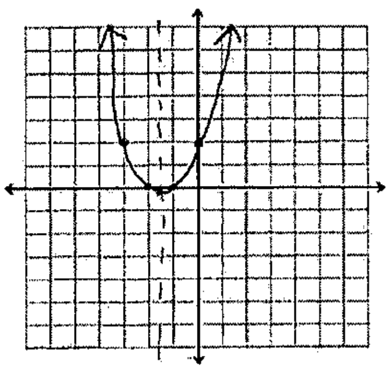
UNIT 4: Factoring and Quadratic Functions (Ch.5)

1. **Sketch the graph of the quadratic function. Identify the vertex, axis of symmetry, and x-intercept(s), domain, range and state the maximum or minimum value.**

(a) $f(x) = x^2 + 3x + 2$



$f(x) = (x+2)(x+1)$

x-int @ $x = -2$, $x = -1$

$(-2, 0)\ (-1, 0)$

y-int @ $y = 2 \rightarrow (0, 2)$

Vertex @ $(-1.5, -0.25)$

$(-3/2, -1/4)$

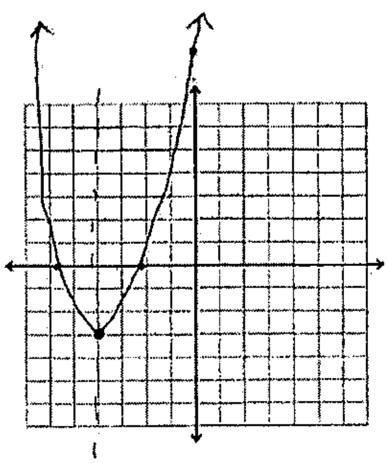
Domain: $x \in \mathbb{R}$

Range: $y \geq -1/4$

Min value: $y = -1/4$

Axis of symmetry: $x = -3/2$

(b) $f(x) = (x+4)^2 - 3$



Vertex: $(-4, -3)$

y-int: $y = 13$ $(0, 13)$

x-int: $(-2.27, 0)\ (-5.73, 0)$

Axis of symmetry $x = -4$

Min value: $y = -3$

Domain: $x \in \mathbb{R}$

Range: $y \geq -3$

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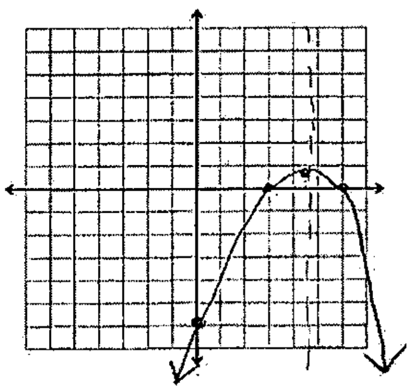
(c) $m(x) = -\frac{1}{3}x^2 + 3x - 6$



y-int: (0,-6)
x-int: (3,0) & (6,0)
Vertex (9/2, 3/4)
Max Value: y = 3/4
Axis of Symmetry: x = 9/2
Domain: $x \in \mathbb{R}$
Range: $y \leq 3/4$

2. **Write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.**

a) Vertex: (-2, 5); Point: (0, 9)

$F(x) = a(x+2)^2 + 5$

$9 = a(0+2)^2 + 5$

$9 = 4a + 5$

$4 = 4a$

$a = 1$

$\boxed{f(x) = 1(x+2)^2 + 5}$

b) Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$; x-intercept = -2 → (-2, 0)

$f(x) = a(x - 5/2)^2 - 3/4$ → Plug in (-2, 0) $0 = a(-2 - \frac{5}{2})^2 - 3/4$

$3/4 = 20.25a$ → $a = 1/27$

$\boxed{f(x) = 1/27(x - 5/2)^2 - 3/4}$

3. **Write the standard form of the quadratic function that passes through the following 3 points.** (0,2),(6,2)(-8,4)

axis of symmetry $= \frac{6-0}{2} = 3$

vertex = (3, k)

$\boxed{f(x) = \frac{1}{56}(x-3)^2 + \frac{103}{56}}$

$f(x) = a(x-3)^2 + k$

$2 = a(-3)^2 + k$

$k = 2 - 9a$ $K = 103/56$

→ $f(x) = a(x-3)^2 + 2 - 9a$

$4 = a(-8-3)^2 + 2 - 9a$

$4 = 112a + 2$ $a = 1/56$

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4. **What is the maximum area of a rectangle that can be constructed with a perimeter of 64 cm?**

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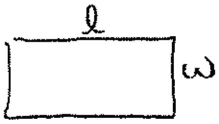

$2l + 2w = 64$

$l = \frac{64 - 2w}{2} \rightarrow l = 32 - w$

Area $\rightarrow A(l) = w(32 - w)$

$A(l) = -w^2 + 32w$

$A(l) = -1(w^2 - 32w + 256) + 256$

$= -1(w - 16)^2 + 256$ ← max area occurs when $w = 16$

$\boxed{\text{The maximum are is } 256\,cm^2}$

UNIT 5: Solving Quadratic Equations (CH. 6)

1) **Solve the following Quadratic Equations (use the method of your choice)**

a. $(x + 13)^2 = 25$

$x + 13 = \pm 5$

$x = 5 - 13 = -8$

$x = -5 - 13 = -18$

$\boxed{x = -8, -18}$

b. $(2x + 3)^2 - 27 = 0$

$(2x + 3)^2 = 27$

$2x + 3 = \pm\sqrt{27}$

$2x + 3 = \pm 3\sqrt{3}$

$2x = -3 \pm 3\sqrt{3}$

x

c. $(x - 7)^2 = (x + 3)^2$

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d. $\frac{1}{8}x^2 - x - 16 = 0$

$\frac{1}{8}(x^2 - 8x + \_\_\_) - 16 = 0$

$\frac{1}{8}(x^2 - 8x + 16) - 16 - 2 = 0$

$\frac{1}{8}(x-4)^2 - 18 = 0$

$(x-4)^2 = 144$

$\sqrt{(x-4)^2} = \pm\sqrt{144}$

$x - 4 = \pm 12$

$x = 4 \pm 12$

$\boxed{x = 16, -8}$

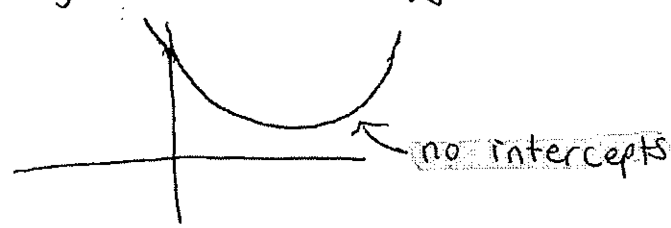
e. $3x^2 + 24x + 16 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-24 \pm \sqrt{24^2 - 4(3)(16)}}{2(3)} = \frac{-24 \pm \sqrt{384}}{6} = \frac{-24 \pm 8\sqrt{}}{6}$

$\boxed{x = -0.73, -7.3}$ $\leftarrow$ OR $\rightarrow$ $\boxed{= \frac{-12 \pm 4\sqrt{6}}{3}}$

f. $\frac{1}{4}x^2 - 2x + 7 = 0$

Solve by graphing: NO SOLUTION



g. $12x - 9x^2 = -3$

$-9x^2 + 12x + 3 = 0$

$-3(3x^2 + 4x + 1) = 0$

$-3(3x+1)(x+1) = 0$

$\boxed{x = -\frac{1}{3}, -1}$

h. $25x^2 + 80x + 61 = 0$

$x = \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)} = \frac{-80 \pm \sqrt{300}}{50} = \frac{-80 \pm 10\sqrt{3}}{50}$

$\boxed{x = \frac{-8 \pm \sqrt{3}}{5} = -1.25, -1.94}$

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i. $3x + 4 = 2x^2 - 7$

$$2x^2 - 3x - 11 = 0$$

* Solved by graphing

$$\boxed{x = -1.71,\ 3.21}$$

a. $2x^2 - 3x = 4x + 12$

$$2x^2 - 7x - 12 = 0$$

$$\boxed{x = 4.76,\ -1.26}$$

2. Brian decides to start training for swimming in a river. The current in the river is 4km/hr. If he swims upstream 2 km and then back downstream to where he started in 3 hours, what is his swimming speed?

$x =$ Brian's swimming speed

	R	D	t
Up	$x-4$	2km	$\frac{2}{x-4}$
Down	$x+4$	2km	$\frac{2}{x+4}$

total time $= 3 = \text{time}_{up} + \text{time}_{down}$

$$\frac{2}{x+4} + \frac{2}{x-4} = 3$$

$$2(x-4) + 2(x+4) = 3(x-4)(x+4)$$

$$2x - 8 + 2x + 8 = 3(x^2 - 16)$$

$$4x = 3x^2 - 48$$

$$3x^2 - 4x - 48 = 0$$

$x = 4.7$ km/hr

Brian's swimming speed is 4.7 km/hr