# Chapter 7 Notes

**STUDENT COPY**

<table>
<thead>
<tr>
<th>Marks Requirement</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notes Present</td>
<td>All notes present</td>
<td>Most notes present</td>
<td>Less than half of notes present</td>
</tr>
<tr>
<td>Organization / Neatness</td>
<td>Notes in chronological order, name and date on everything</td>
<td>Almost all notes in chronological order, name and date on most pages</td>
<td>Mostly out of order, name and date often missing</td>
</tr>
<tr>
<td>Questions</td>
<td>Question column completed on all notes, higher level questions attempted</td>
<td>Most question columns complete, some higher level questions</td>
<td>Less than half of the question columns complete</td>
</tr>
<tr>
<td>Main Ideas and Reflections</td>
<td>All ‘main ideas’ and ‘reflections’ complete with care in notes</td>
<td>Most ‘main ideas’ and ‘reflections’ complete in notes</td>
<td>Less than half of the ‘main ideas’ and ‘reflections’ complete</td>
</tr>
</tbody>
</table>

*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

**TEACHER COPY**

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7.2 – Solving Systems with Graphs

Goals: to use graphs to solve linear systems

Toolkit:
- graphing lines
- rearranging equations
- substitution

Definitions:

Linear System – two or more linear equations together is called a **linear system**.

Solving a System – to solve a linear system, find the coordinates where the two lines intersect (the point where the lines cross). You will have an x-value and a y-value.

Steps for solving systems graphically:

1. Change each equation to a form that is easy to graph (\( y = mx + b \) or \( Ax + By = C \))
2. Graph each line
3. Write the solution (state the point where the lines cross) \((x, y)\)
4. Check the solution by substituting into each original equation (point must “satisfy” both lines)

<table>
<thead>
<tr>
<th>What are the three possibilities when two lines are graphed?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong> One solution</td>
</tr>
<tr>
<td><img src="image1" alt="Graphs" /></td>
</tr>
</tbody>
</table>

**Ex1)** Solve the system graphically and check the solution:

\[
\begin{align*}
&\begin{align*}
1. & \quad x + y = 7 \\
2. & \quad 3x + 4y = 24
\end{align*}
\end{align*}
\]

**Step 1:**
- \( x + y = 7 \)  
- \( x + 7 \)  
- \( y = 7 \)  
- \( y = \frac{-1}{1} \) 
- \( \text{Slope:} \)
- \( y = -\frac{1}{1} \) 
- \( y = \frac{7}{1} \)

**Step 2:**
- \( 3x + 4y = 24 \)  
- \( x = 8 \)  
- \( y = 6 \)  
- \( \text{Slope:} \)
- \( y = -\frac{3}{4} \) 
- \( y = \frac{3}{4} \) 
- \( y = \frac{6}{4} \)

**Step 3:** Solution to \((4,3)\)

**Step 4:**
- Check:
  - \( x + y = 7 \) 
  - \( 4 + 3 = 7 \)
  - \( 3x + 4y = 24 \) 
  - \( 3(4) + 4(3) = 24 \)
Ex2) Is \((2, -1)\) a solution to the following system?

Check:

\[
\begin{array}{c|c|c}
1 & 2.5 & 2.5 \\
\frac{3x + 5y}{3(2) + 5(-1)} & \frac{2x - 2y}{2(2) - 2(-1)} & \frac{6}{4 + 2} \\
6 + 5 & 2 & 6 \\
1 & & \\
\end{array}
\]

\((2, -1)\) does not satisfy both equations.

\(\therefore \) is not a solution!

Ex3) Solve the system by graphing. Explain whether the solution is exact or approximate.

Step 1:

1. \(x + 2y - 5 = 0 \Rightarrow 0x + 2y = 5\)
2. \(x - 2y - 13 = 0 \Rightarrow 0x - 2y = 13\)

Step 2:

1. \(x + 2y = 5\)
   \(\Rightarrow \frac{x + 2y}{2} = \frac{5}{2}\) slope: \(-\frac{1}{2}\)
2. \(x - 2y = 13\)
   \(\Rightarrow \frac{x - 2y}{2} = \frac{13}{2}\) slope: \(\frac{1}{2}\)

Step 3: Solution is \((9, -2)\)

Step 4:

1. \(x + 2y = 5\)
2. \(x + 2y = 13\)

The solution is exact (but was hard to be sure)

Ex4) Solve the system by graphing. Explain whether the solution is exact or approximate.

1. \(y = -2x + 7\)
2. \(7x - 2y = 0\)

Step 1:

1. \(y = -2x + 7\)
2. \(7x - 2y = 0\)

Step 2:

1. \(m = \frac{-2}{1} = -2\)
2. \(m = \frac{7}{2} = \frac{7}{2}\)

Step 3: Solution is \((1, 3, 4.2)\)

Step 4: The solution is approximate.

Reflection: What is a disadvantage to solving a system using GRAPHING?

- The results may be approximate or hard to see.
- It is difficult with large or awkward numbers.
7.4 – Solving Systems Algebraically using Substitution

**Goal:** to use the substitution of one variable to solve a linear system

**Toolkit:**
- Rearranging equations
- Substituting into eqns
- A solution is the point \((x, y)\)
  where the lines meet. \(x\) need \(y\)!

**Main Ideas:**

Linear systems can be solved without graphing. One method is by substitution.

**Steps:**
1. Solve one equation for either \(x\) or \(y\) (get either \(x\) or \(y\) by itself).
   Let's say you get \(y\) by itself in this case.
2. Substitute the equation into the second equation
3. Solve the second equation for the other variable (in this case \(x\))
4. Now that you have the solution to one variable (in this case \(x\)), substitute it into one of the original two equations to get \(y\).
5. Write the solution
6. Check that the solution satisfies each equation

- **Ex 1:** Solve by substitution and check

  \[
  \begin{align*}
  3x + y &= 3 \\ 7x - 2y &= 20
  \end{align*}
  \]

  **Step 1:** Look at both equations. Which variable, in which eqn is easiest to get by itself? \(y\) in \(1\).

  **Step 2:** \( 7x - 2y = 20 \)

  Step 2: \( 7x - 2(-3x + 3) = 20 \)

  \( 7x + 6x - 6 = 20 \)

  \[
  \begin{align*}
  13x &= 26 \\
  x &= 2
  \end{align*}
  \]

  **Step 3:** Substitute \(x = 2\) in either original equation

  **Step 4:** Solve the equations

  \[
  \begin{align*}
  3x + y &= 3 \\
  3(2) + y &= 3 \\
  6 + y &= 3 \\
  y &= -3
  \end{align*}
  \]

  **Step 5:** If these lines were graphed, they would intersect at \((2, -3)\).

  **Solution:** \((2, -3)\)

  **Step 6:** Check!

  \[
  \begin{array}{c|c|c}
  \hline
  \text{LS} & \text{ES} & \text{LS} \\
  \hline
  3x + y & 3 & 20 \\
  7x - 2y & 4 & 26 \\
  \hline
  \end{array}
  \]

  \[
  \begin{align*}
  x &= 2 \\
  y &= -3
  \end{align*}
  \]
<table>
<thead>
<tr>
<th>Equations solved for ( y )</th>
<th>Ex2) Solve by substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{y}{2} = \frac{2x + 2}{x - 14} )</td>
<td><strong>Step 1</strong>: ( y ) already by itself in ( ① )</td>
</tr>
<tr>
<td>( 3x + 2 = -x - 14 )</td>
<td>( y = 3x + 2 )</td>
</tr>
<tr>
<td>( 3x + z = -16 )</td>
<td>( y = 3(-4) + 2 )</td>
</tr>
<tr>
<td>( z )</td>
<td>( y = -12 + 2 )</td>
</tr>
<tr>
<td>( y = -10 )</td>
<td><strong>Solution</strong>: ((-4, -10)) or ( x = -4, y = -10 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equations with fractions</th>
<th>Ex3) Solve by substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Clear fractions!</em></td>
<td>Clear Fractions!</td>
</tr>
<tr>
<td><em>Clear fractions!</em></td>
<td>multiply all terms by a number (LCM) that will cancel with denominators.</td>
</tr>
<tr>
<td>( \frac{x}{2} + \frac{y}{3} = 0 )</td>
<td>( 0 - 3x + 5y - 2 = 0 )</td>
</tr>
<tr>
<td>( \frac{x}{2} + \frac{y}{3} = 0 )</td>
<td>( x + 7y = 0 )</td>
</tr>
<tr>
<td>( x = -7y )</td>
<td><strong>Solution</strong>: ( (-\frac{7}{13}, \frac{1}{13}) ) or ( x = -\frac{7}{13}, y = \frac{1}{13} )</td>
</tr>
</tbody>
</table>

**Reflection**: When you have a system with fractions in it, and you want to write an equivalent system without fractions, how do you decide what number to multiply by?
7.5 – Solving Systems Algebraically using Elimination

Goal: to use the elimination of one variable to solve a linear system

Toolkit:
- Substitution
- Rearranging eqns
- Coefficient: the number multiplied by a variable
e.g. \( 3x \rightarrow \) coefficient of \( x \) is 3.

Main Ideas:
Linear systems can be solved without graphing. One method is by elimination.

Steps:
1. *May not be necessary* Multiply both sides of one or both equations by a constant to get either the same \( x \) or the same \( y \) coefficient in both equations *you get an “equivalent system”.
2. Add or subtract the two equations to eliminate either \( x \) or \( y \)
3. Solve the resulting equation for the remaining variable
4. Substitute the value obtained in step 3 back into one of the original equations to get the other variable
5. Write the solution
6. Check that the solution satisfies each equation

Ex 1) Solve the system by elimination and check

\[
\begin{align*}
3x - 5y &= -9 \\
4x + 5y &= 23
\end{align*}
\]

\[
\begin{align*}
\text{Step 1:} & \quad \text{not needed (the \( y \)'s can be eliminated by adding the eqns.)} \\
\text{Step 2:} & \quad 7x = 14 \\
\text{Step 3:} & \quad x = 2 \\
\text{Step 4:} & \quad \text{sub in } x = 2 \text{ to either eqn } \{1 \text{ or } 2\}
\end{align*}
\]

\[
\begin{align*}
3x - 5y &= -9 \\
4x + 5y &= 23
\end{align*}
\]

\[
\begin{align*}
3(2) - 5y &= -9 \\
6 - 5y &= -9 \\
-5y &= -15 \\
y &= 3
\end{align*}
\]

\[
\begin{align*}
3x - 5y &= -9 \\
4x + 5y &= 23
\end{align*}
\]

\[
\begin{align*}
15 - 5y &= -9 \\
15 - 5y &= -9
\end{align*}
\]

If the variable you are eliminating has the same coefficient (but with different signs), you must add.
e.g. \( -5y \)

If the coefficients are exactly the same (same sign, too), you subtract.
e.g. \( +2y \)
Ex 2) Solve by elimination: \[
\begin{align*}
4x + 3y &= 5 \\
4x - 7y &= 15
\end{align*}
\]

\[
\begin{align*}
10x + 3y &= 5 \\
10x - 7y &= 15
\end{align*}
\]

\[
\begin{align*}
y &= -1 \\
x &= 2
\end{align*}
\]

Solution: \((2, -1)\) or \(x = 2, y = -1\)

Ex 3) Solve by elimination: \[
\begin{align*}
2x + 5y &= 11 \\
3x - 2y &= 7
\end{align*}
\]

Clear fractions: \[
\begin{align*}
6x + 15y &= 33 \\
6x - 4y &= 14
\end{align*}
\]

\[
\begin{align*}
y &= 1 \\
x &= 3
\end{align*}
\]

Solution: \((3, 1)\)

Ex 4) Solve by elimination: \[
\begin{align*}
\frac{3x}{4} - \frac{y}{2} = 2 \\
\frac{10x}{3} + \frac{10y}{3} = 8
\end{align*}
\]

Clear fractions: \[
\begin{align*}
3x - 4y &= 8 \\
10x + 10y &= 24
\end{align*}
\]

\[
\begin{align*}
x &= 40 \\
y &= 5
\end{align*}
\]

Solution: \((8, 4)\)

Reflection: Which method do you prefer for solving linear systems AND WHY: graphing, substitution, or elimination?
Goal: to recognize systems that will have each of the three different types of solutions

Toolkit:
- So far, all of the linear systems we've solved have given one solution (one intersection)
- Rearranging equations

Main Ideas:

<table>
<thead>
<tr>
<th>Three types of solutions:</th>
<th>Sketch:</th>
<th>Description:</th>
<th>Check Slopes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A  ONE solution</td>
<td>![Sketch A]</td>
<td>different slopes</td>
<td>different slopes → same slope? check y-int:</td>
</tr>
<tr>
<td>B  NO solution</td>
<td>![Sketch B]</td>
<td>same slope</td>
<td>same y-int ↓</td>
</tr>
<tr>
<td>C  Infinite solutions</td>
<td>![Sketch C]</td>
<td>same slope</td>
<td>same y-int ↓</td>
</tr>
</tbody>
</table>

How can you predict how many solutions a system will have without graphing?

Ex1) Predict how many solutions each system has:

a) \[y = 2x + 3\]
   \[y = \frac{6}{3} x + 3\]
   
   \[m = 2, b = 3\]

b) \[\begin{array}{l}
   y - x = 4 \\
   2x + y = 5
\end{array}\]

   \[m = \begin{cases}
   +1 & \text{different slopes} \\
   -1 & \text{same slope}
\end{cases}\]

   \[\text{different y-int} \downarrow\]

   \[\therefore \text{one solution}\]

c) \[\begin{array}{l}
   y = \frac{-1}{2} x + 7 \\
   y = \frac{-1}{2} x + 2
\end{array}\]

   \[\text{same slope}
   \begin{cases}
   \text{different} & \text{y-int.} \\
   \text{same} & \text{y-int.}
\end{cases}\]

   \[\therefore \text{no solution}\]
Standard form “shortcut”: start off like elimination—try to get $x$ or $y$ coefficients to match by multiplying the whole equation by a constant

Ex 2) How many solutions?

\[
\begin{align*}
2x - 5y &= 15 \\
4x - 10y &= 30 \\ 4x - 10y &= 6
\end{align*}
\]

\[\checkmark \checkmark \checkmark \quad \text{not a match} \]

\[\checkmark \checkmark \quad \text{match match No solution!} \]

A) If $x$ and $y$ coefficients DO NOT BOTH match, then you have ONE solution

B) If $x$ and $y$ coefficients BOTH match, but the constants DO NOT, then you have NO solution

C) If $x$ and $y$ coefficients BOTH match, and the constants match, then you have INFINITE solutions (all 3 numbers are the same)

Ex 3) How many solutions does each system have?

a) \[
\begin{align*}
7x - y &= 10 \\
14x - 2y &= 20
\end{align*}
\]

\[\begin{align*}
\times 2 & \quad 14x - 2y = 20 \\
\rightarrow & \quad 14x - 2y = 20 \\
\checkmark & \quad \checkmark \quad \checkmark
\end{align*}
\]

all 3 match (same slope, same y-inter)

\[\therefore \quad \text{inf solutions} \]

b) \[
\begin{align*}
4x - 3y &= 12 \\
8x - 6y &= 30
\end{align*}
\]

\[\begin{align*}
\times 2 & \quad 8x - 6y = 24 \\
\rightarrow & \quad 8x - 6y = 30 \\
\checkmark & \quad \checkmark \quad x
\end{align*}
\]

$x$- and $y$- match (same slope) but constant doesn't justify

\[\therefore \quad \text{no solution} \]

c) \[
\begin{align*}
5x + y &= 16 \\
2x - 3y &= 3
\end{align*}
\]

\[\begin{align*}
\times -3 & \quad -15x + 3y = -48 \\
\rightarrow & \quad 2x - 3y = 3 \\
& \quad \checkmark \quad x
\end{align*}
\]

different $x$-coefficients (different slopes)

\[\therefore \quad \text{one solution} \]

Reflection: Use examples and/or diagrams to explain why there cannot be exactly 2 solutions to a linear system.

To cross only 2 times, a curve would be needed, then it would not be linear anymore.
Goal: to model situations and answer problems using a system of linear equations

Toolkit:
- total
- sum, greater than is +
- difference, less than is –
- times, product is ×
- to change % to dec, move decimal two places to the left
- remember units!

Main Ideas:

These word problems involve two unknowns. We need two equations to solve for two unknowns, so it will be your job to create the system of two equations and solve it!

Steps:
1. Define your two variables. You may use x and y, but it is also good to practice working with other variables (such as t for time). Use “let” statements (e.g. let x be the number of…)
   Usually, they are the two things you need in order to answer the problem.
2. Build your two equations.
3. Solve the system using elimination, substitution, or graphing.
4. Write a sentence answer.
5. Check.

Example:

Ex 1) The sum of two numbers is 53. The first is 7 greater than the second.

What are the numbers?

Step 1: Let x be the “first number.” Let y be the “second number.”

Step 2: From sentence 1: sum (+) of #s = x, y (→) 53.
   1) x + y = 53

   From sentence 2: first (x) is (=) 7 more (+7) than second (y)
   2) x = y + 7

Step 3: Solve! Use substitution (since eqn (2) already solved for x…)

\[ x = y + 7 \]

\[ y + 7 + y = 53 \]
\[ 2y + 7 = 53 \]
\[ 2y = 46 \]
\[ y = 23 \]

Step 4: The two numbers are 30 and 23.

Step 5: Is 30 + 23 more than 23? Yes!
Is 30 + 23 equal to 53? Yes!
Ex 2) For a basketball game, 1600 tickets were sold. Some tickets cost $3 and the rest cost $2. If the total receipts were $4000, how many of each kind were sold?

1. Let $x =$ # of 3-dollar tix  
   Let $y =$ # of 2-dollar tix.

2. "1600 tickets sold" $\rightarrow$ \( x + y = 1600 \) (check units \( \text{tix} + \text{tix} = \text{tix} \))

   "total receipts $4000" \rightarrow \text{money} + \text{money} = \text{money}:

   \[ 3x + 2y = 4000 \]

3. Solve: try elimination

\[
\begin{align*}
  x + y &= 1600 \\
  2x + 2y &= 3200 \\
  3x + 2y &= 4000
\end{align*}
\]

Subtract the top equation from the middle and the middle from the bottom:

\[
\begin{align*}
  2x + 2y - (x + y) &= 3200 - 1600 \\
  3x + 2y - (2x + 2y) &= 4000 - 3200 \\
  x &= 800 \\
  y &= 800
\end{align*}
\]

Therefore, there were 800 3-dollar tickets and 800 2-dollar tickets sold.

Ex 3) Isaac borrowed $2100 for his college tuition. Part of it he borrowed from a government student fund at 5% annual interest. The rest he borrowed from a bank at 6.5% annual interest. If the total annual interest is $114, how much did he borrow from each source?

Let $g =$ amount from govt'  
Let $b =$ amount from bank.

\[ g + b = 2100 \]

\[ 0.05g + 0.065b = 114 \]

3. Elimination \( \rightarrow \) big $\#$! Try subst.

\[
\begin{align*}
  g &= 2100 - b \\
  0.05(2100 - b) + 0.065b &= 114 \\
  105 - 0.05b + 0.065b &= 114 \\
  105 + 0.015b &= 114 \\
  0.015b &= 9 \\
  b &= 600
\end{align*}
\]

Isaac borrowed $1500 from the government and $600 from the bank.

**Reflection:** Would you ever need to solve for 3 variables? Think of a scenario and (no need to solve!) explain WHAT you would need in order to be able to solve for 3 variables.
Goal: to continue to model situations and answer problems using a system of linear equations.

Toolkit:
- Sum (+), difference (−), product (×)
- To change % to dec, move decimal two places to the left
- Speed = \( \frac{\text{dist.}}{\text{time}} \)
- (tv in the basement)
- Remember units

Main Ideas:

These word problems involve two unknowns. We need two equations to solve for two unknowns, so it will be your job to create the system of two equations and solve it!

Steps:
1. Define your two variables. You may use \( x \) and \( y \), but it is also good to practise working with other variables (such as \( t \) for time). Use “let” statements (e.g. let \( x \) be the number of…).
   Usually, they are the two things you need in order to answer the problem.
2. Build your two equations.
3. Solve the system using elimination, substitution, or graphing.
4. Write a sentence answer.
5. Check.

Ex 1) The perimeter of a rectangle is 46 cm. What are its dimensions if the length is 4 cm less than twice the width?

\[
P = 2l + 2w
\]

\[
\text{Let } l = \text{length and } w = \text{width.}
\]

1. \[2l + 2w = 46\] (note: \( \text{cm} \div 2 \rightarrow l + w = 23\))

2. \[l = 2w - 4\]
   \[\text{"length is" \ twice \ "width" \ "4 less than"}\]

3. \[l = 2w - 4\]
   \[0l + w = 23\]
   \[1w - 4 + w = 23\]
   \[3w - 4 = 23\]
   \[3w = 27\]
   \[w = 9\]

\[
l = 2w - 4
\]
\[
l = 2(9) - 4
\]
\[
l = 18 - 4
\]
\[
l = 14
\]

4. The dimensions are 9 cm by 14 cm.

3. Check: \[P = 2 \times 9 + 2 \times 14\]
   \[= 18 + 28\]
   \[= 46 \checkmark\]
Ex 2) Flying with the wind, an airplane travels 4256 km in 3.5 h. Flying against the same wind, the airplane makes the return trip in 3.8 h. Find the speed of the airplane in still air and the speed of the wind (assume both speeds are constant for the round trip).

Whenever you're doing a word problem with speed, distance, and time, it helps to set up a table like the one below:

Let \( a = \text{speed of airplane} \)

Let \( w = \text{speed of wind} \)

<table>
<thead>
<tr>
<th>Direction</th>
<th>Distance (km)</th>
<th>Speed (km/h)</th>
<th>Time (h)</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>With the wind (add)</td>
<td>4256</td>
<td>( a + w )</td>
<td>3.5</td>
<td>( a + w = \frac{4256}{3.5} )</td>
</tr>
<tr>
<td>Against the wind (subtract)</td>
<td>4256</td>
<td>( a - w )</td>
<td>3.8</td>
<td>( a - w = \frac{4256}{3.8} )</td>
</tr>
</tbody>
</table>

Equations from last column:

1. \( a + w = \frac{4256}{3.5} \) \( \rightarrow \) \( a + w = 1216 \)
2. \( a - w = \frac{4256}{3.8} \) \( \rightarrow \) \( a - w = 1120 \)

Elimination: \( 2a = 2336 \)

Sub back in:

\( a + w = 1216 \)

\( 1168 + w = 1216 \)

\( w = 48 \)

The speed of the airplane (in still air) was 1168 km/h and the speed of the wind was 48 km/h.

Reflection: How will YOU remember the relationship among distance, speed, and time?

1. "Triangulate"
2. Speed = dist/time
3. Memorize!
4. Others?