# Chapter 4 Notes

**STUDENT COPY**

<table>
<thead>
<tr>
<th>Marks Requirement</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notes Present</td>
<td>All notes present</td>
<td>Most notes present</td>
<td>Less than half of notes present</td>
</tr>
<tr>
<td>Organization / Neatness</td>
<td>Notes in chronological order, name and date on everything</td>
<td>Almost all notes in chronological order, name and date on most pages</td>
<td>Mostly out of order, name and date often missing</td>
</tr>
<tr>
<td>Questions</td>
<td>Question column completed on all notes, higher level questions attempted</td>
<td>Most question columns complete, some higher level questions</td>
<td>Less than half of the question columns complete</td>
</tr>
<tr>
<td>Main Ideas and Reflections</td>
<td>All 'main ideas' and 'reflections' complete with care in notes</td>
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<td>Less than half of the 'main ideas' and 'reflections' complete</td>
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*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

**TEACHER COPY**

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4.1 – Estimating Roots

Goal: to explore decimal representations of different roots of numbers

Toolkit:
- Finding a square root
- Finding a cube root
- Multiplication
- Estimating

Main Ideas:

Definitions:
Radical: an expression consisting of a radical sign, a radicand, and an index.

Perfect squares and cubes to memorize:
- \( \sqrt{4} = 2 \), \( \sqrt{9} = 3 \), \( \sqrt{16} = 4 \), \( \sqrt{25} = 5 \), \( \sqrt{36} = 6 \)
- \( \sqrt{49} = 7 \), \( \sqrt{64} = 8 \), \( \sqrt{81} = 9 \), \( \sqrt[3]{8} = 2 \), \( \sqrt[3]{27} = 3 \), \( \sqrt[3]{64} = 4 \), \( \sqrt[3]{125} = 5 \)

Ex 1) Evaluate the following radicals, identify the radicand and index for each:

a) \( \sqrt{16} = 4 \)
   
   \[ 4 \times 4 = 16 \]
   
   Radicand: \( 16 \)
   
   Index: \( 2 \)

b) \( \sqrt[3]{64} = 4 \)
   
   \[ 4 \times 4 \times 4 = 64 \]
   
   Radicand: \( 64 \)
   
   Index: \( 3 \)

* if no index is written the index is a 2.

Ex 2) Estimate the value of \( \sqrt{20} \) to one decimal place.

Step 1: Find the two perfect squares that are closest to the radicand you are looking for (one that is lower and one that is higher).

\[ \sqrt{16} \quad \sqrt{25} \]

\[ \sqrt{20} \]

Step 2: Find which of the two perfect squares is closest to your radicand; this will determine the decimal point of your root.

\( 20 - 16 = 4 \) \( 20 \) is closer to \( 16 \) so the root is closer to 4.

\( 25 - 20 = 5 \) \( 20 \) is closer to 5.

\( \sqrt{20} \approx 4.4 \)

Evaluate \( \sqrt{20} \), how close was your estimate?

\( \sqrt{20} = 4.47 \)
### Estimating Cube Roots

**Ex 3)** Estimate the value of \( \sqrt[3]{16} \)

**Step 1:** Find the two perfect cubes that are closest to the radicand you are looking for.

\[
\begin{align*}
3^3 & = 27 \\
2^3 & = 8
\end{align*}
\]

**Step 2:** Find which of the two perfect cubes is closest to your radicand.

\[
16 - 8 = 8 \rightarrow \sqrt[3]{16} \text{ is closest to } \sqrt[3]{8}.
\]

\[
\sqrt[3]{16} \approx 2.4.
\]

Evaluate \( \sqrt[3]{16} \), how close was your estimate?

\[
\sqrt[3]{16} \approx 2.5.
\]

Why can you take the cube root of a negative number but not the square root of a negative number?

\[
0.64 = \frac{64}{100} \quad \frac{\sqrt{64}}{\sqrt{100}} = \frac{8}{10} = 0.8
\]

\[
0.0196 = \frac{196}{10000} \quad \frac{\sqrt{196}}{\sqrt{10000}} = \frac{14}{100} = 0.14
\]

**Ex 4)** Estimate the value of \( \sqrt[3]{-32} \)

\[
\sqrt[3]{-32} \quad \sqrt[3]{-32} \text{ is significantly closer to } -\frac{3}{2}, -4, -8 \quad \sqrt[3]{-32} \approx -3.2
\]

**Ex 5)** Evaluate \( \sqrt{0.64} \)

*If radicand has 2 decimal places, then root has one decimal place*

We know \( \sqrt{64} = 8 \)

So, \( \sqrt{0.64} = 0.8 \)

**Ex 6)** Evaluate \( \sqrt{0.0196} \)

*If radicand has 4 decimal places, then root has two.*

We know \( \sqrt{196} = 14 \)

So, \( \sqrt{0.0196} = 0.14 \)

**Ex 7)** Write an equivalent form of 0.3 as a cube root.

\[
0.3 \times 0.3 \times 0.3 = 0.03 \Rightarrow \sqrt[3]{0.03} = 0.3
\]

**Reflection:** How would you write 5 as a square root? A cube root? A fourth root?

\[
\begin{align*}
\text{Square root} & = \sqrt{25} = 5 \\
\text{Cube root} & = \sqrt[3]{125} = 5 \\
\text{Fourth root} & = \sqrt[4]{625} = 5
\end{align*}
\]
4.2 – Irrational Numbers

Goal: to classify real numbers, and to identify & order irrational numbers

Toolkit:
- Estimating roots
- Placing numbers on number lines
- Anything you remember about classifying Real Numbers

Main Ideas:

Natural Numbers (\( \mathbb{N} \)) Counting numbers:

Whole Numbers (\( \mathbb{W} \)) 0 & counting numbers:

Integers (\( \mathbb{Z} \)) negative, counting, zero, positive:

Rational Numbers (\( \mathbb{Q} \)) can be written as \( \frac{m}{n} \) integers (n≠0)

Decimals: repeating \( 0.\overline{3} \), \( \frac{1}{3} \)
or terminating (ending - 0.25 \( \frac{1}{4} \))

Irrational Numbers (\( \mathbb{Q}^c \)) Not Rational! Cannot be written as a fraction;
Decimals do not repeat or terminate. (\( \sqrt{2}, \pi \))

Ex1) Where do these numbers belong in the diagram of Real numbers?

\[
\begin{align*}
2 & \quad 0.\overline{6} & \quad 4\sqrt{2} & \quad \frac{3}{4} & \quad -\frac{8}{2} & \quad -12 & \quad \pi & \quad 0 & \quad \sqrt{16} = 4 \\
\frac{2}{3} & \quad 1.35 & \quad -\sqrt{5} & \quad \sqrt{3} & \quad \sqrt{15} & \quad 19 & \quad \frac{4}{9} & \quad \frac{2}{3}
\end{align*}
\]

Real Numbers: Sticks " terminates"

Rational Numbers (\( \mathbb{Q} \))

<table>
<thead>
<tr>
<th>Whole Numbers (( \mathbb{W} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Numbers (( \mathbb{W} ))</td>
</tr>
<tr>
<td>2 \quad \sqrt{16} = 4</td>
</tr>
<tr>
<td>( \frac{2}{3} ) \quad \sqrt{\frac{125}{3}} = 5</td>
</tr>
<tr>
<td>19</td>
</tr>
</tbody>
</table>

Irrational Numbers (\( \mathbb{Q}^c \))

| \( \sqrt{2} \) |
| \( \sqrt{3} \) |
| \( \sqrt{15} \) |
Ex2) Use a number line to order these numbers from least to greatest.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\sqrt{6})</td>
<td>(-\sqrt{2})</td>
<td>(\sqrt{11})</td>
<td>(\sqrt{30})</td>
<td>(\sqrt{2})</td>
</tr>
</tbody>
</table>

Between \(\sqrt{1}\) and \(\sqrt{8}\): 
- \(\sqrt{1} = 1\)
- \(\sqrt{8} = 2\)

Or use calc: 
- \(\sqrt{6} \approx 2.449489743\) (or, between
  - \(\sqrt{4} = 2\)
  - \(\sqrt{9} = 3\)

Ex3) Is the tangent ratio for \(\theta\) in each right triangle rational or irrational?

**a)**
- \(\tan \theta = \frac{\text{opp}}{\text{adj}}\)
- \(\tan \theta = \frac{3}{4}\) \(\rightarrow\) Rational!

**b)**
- \(\tan \theta = \frac{\text{opp}}{\text{adj}}\)
- \(\tan \theta = \frac{\sqrt{3}}{1}\) \(\rightarrow\) Irrational!

Reflection: How could you order a set of irrational numbers if you do not have a calculator?

Estimate all of the decimals, then place on a number line.
4.3A – From Entire to Mixed Radicals

Goal: to express an entire radical as a mixed radical

Toolkit:
- Understanding Radicals
- Identifying Factors of a Number

Main Ideas:

Perfect Squares - 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ....

Perfect Cubes - 1, 8, 27, 64, 125, 216, ....

A radical sign with a number under it: \( \sqrt[3]{8}, \sqrt[3]{64} \)

A number written as the product of a number and a radical: \( \sqrt[3]{15}, 4\sqrt[3]{10} \)

Equivalent Forms:
Ex. 1)

<table>
<thead>
<tr>
<th>a) ( \sqrt{16} \cdot 9 ) is equivalent to ( \sqrt{16} \cdot \sqrt{9} ) because:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{144} = 12 )</td>
</tr>
<tr>
<td>( \sqrt{216} = 6 )</td>
</tr>
<tr>
<td>( \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} ), where ( a ) is a natural number, and ( a ) and ( b ) are real numbers</td>
</tr>
</tbody>
</table>

What is an entire radical?

What is a mixed radical?

What is the Multiplication Property of Radicals?

Simplifying Square Roots

We can simplify \( \sqrt{24} \) because 24 has a perfect square factor of \( 4 \). (Hint: Look at list of perfect squares)

- Re-write \( \sqrt{24} \) as a product of two factors, with the first one being the perfect square:
  \[ \sqrt{24} = \sqrt{4} \cdot \sqrt{6} \]
  \[ = 2 \sqrt[3]{6} \]

Simplifying Cube Roots

We can also simplify \( \sqrt[3]{24} \) because 24 has a perfect cube factor of \( 8 \). (Hint: Look at list of perfect cubes)

- Re-write \( \sqrt[3]{24} \) as a product of two factors, with the first one being the perfect cube:
  \[ \sqrt[3]{24} = \sqrt[3]{8} \cdot \sqrt[3]{3} \]
  \[ = 2 \sqrt[3]{3} \]
Reflection: How do you use the index of a radical when you simplify a radical? Use an example.

\[ \sqrt[3]{16} \to \text{index } 3, \text{ tells us to look for a perfect square factor} \]

\[ 3\sqrt[3]{2} \to \text{but } \sqrt[3]{8} = 2 \text{ is written } 4 \times \]

\[ V = 128 \text{ cm}^3 \]

\[ V_{\text{cube}} = e^3 \Rightarrow e = \sqrt[3]{V_{\text{cube}}} = \sqrt[3]{128} \]

\[ e = 4.33 \times 2 = 8.66 \text{ cm} \]

Wt. the edge length of the cube

\[ \text{prime factorization of } 128 = 2^7 \]

\[ \text{factor of } 128 = 8 \times 16 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \]

Ex. 5: A cube has a volume of 128 cm³. Write the edge length of the cube in simplest radical form.

Ex. 4: Simplify \( \sqrt[3]{24} \)

Ex. 3: Simplify \( \sqrt[6]{32} \)

Ex. 2: Simplify each radical. (Remember you’ll find perfect squares and perfect cubes)

1. \( \sqrt[4]{64} \)
2. \( \sqrt[4]{128} \)
3. \( \sqrt[4]{256} \)
4. \( \sqrt[4]{1000} \)

Prime factorization of 128:

Since \( \sqrt[4]{128} \) is a fourth root, look for a factor that appears 4 times:

128 = 2^7

Factors:

1. 2
2. 4
3. 8
4. 16
5. 32
6. 64
7. 128

**Note:** Simplify \( \sqrt[4]{128} \) to \( 2 \sqrt[4]{2} \).
4.3B – From Mixed to Entire Radicals

Goal: to express a mixed radical as an entire radical

Toolkit:
- List of Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100...
- List of Perfect Cubes: 1, 8, 27, 64, 125, 216, ...
- Multiplication Property of Radicals \( \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \)
- Mixed Radical...ex. \( \sqrt[3]{7}, 2\sqrt{5}, 6\sqrt[4]{7} \)
- Entire Radical.....ex. \( \sqrt{105}, \sqrt{3} \)

Main Ideas:

Write the mixed radical \( 4\sqrt{3} \) as an entire radical:

\[
4\sqrt{3} = 4 \cdot \sqrt{3} \\
= \sqrt{16} \cdot \sqrt{3} \quad \text{(re-write 4 as a radical.....think .....4 = \sqrt{16})} \\
= \sqrt{16 \cdot 3} \quad \text{- Combine these under the same radical sign and multiply} \\
= \sqrt{48} \quad \text{(****NOTICE...these are the opposite steps to writing an entire radical as a mixed radical)}
\]

Ex. 1) Write each as an entire radical:

<table>
<thead>
<tr>
<th>( \sqrt[5]{25} )</th>
<th>( \sqrt[3]{27} )</th>
<th>( \sqrt[2]{8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5 \cdot \sqrt[5]{5} )</td>
<td>( 3 \cdot \sqrt[3]{3} )</td>
<td>( 2 \cdot \sqrt[2]{2} )</td>
</tr>
<tr>
<td>( = 5 \cdot \sqrt[5]{5} )</td>
<td>( = 3 \cdot \sqrt[3]{3} )</td>
<td>( = 2 \cdot \sqrt[2]{2} )</td>
</tr>
<tr>
<td>( = \sqrt[5]{25 \cdot 5} )</td>
<td>( = \sqrt[3]{27 \cdot 3} )</td>
<td>( = \sqrt[2]{8 \cdot 2} )</td>
</tr>
<tr>
<td>( = \sqrt[5]{125} )</td>
<td>( = \sqrt[3]{81} )</td>
<td>( = \sqrt[2]{16} )</td>
</tr>
<tr>
<td>( = \sqrt[5]{5} \cdot \sqrt[5]{5} )</td>
<td>( = \sqrt[3]{3} \cdot \sqrt[3]{3} )</td>
<td>( = \sqrt[2]{2} \cdot \sqrt[2]{2} )</td>
</tr>
<tr>
<td>( = \sqrt[5]{5} )</td>
<td>( = \sqrt[3]{3} )</td>
<td>( = \sqrt[2]{2} )</td>
</tr>
<tr>
<td>( = \sqrt[5]{125} )</td>
<td>( = \sqrt[3]{27} )</td>
<td>( = \sqrt[2]{8} )</td>
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Write \( 3\sqrt[5]{2} \) as an entire radical:

First, re-write 3 as \( \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \sqrt[5]{243} \)

So now, \[ 3\sqrt[5]{2} = \frac{5 \text{ thees}}{\text{since index is 5!}} \]

\[
= \frac{3 \cdot 5 \sqrt[5]{2}}{\sqrt[5]{243} \cdot \sqrt[5]{2}} \\
= \frac{\sqrt[5]{243} \cdot \sqrt[5]{2}}{\sqrt[5]{243} \cdot 2} \\
= \frac{\sqrt[5]{243 \cdot 2}}{\sqrt[5]{243 \cdot 2}} \\
= \frac{\sqrt[5]{486}}{2}
\]

Ex. 2) Write each as an entire radical:

<table>
<thead>
<tr>
<th>( \sqrt[4]{5} )</th>
<th>( \sqrt[4]{2} )</th>
</tr>
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<tbody>
<tr>
<td>( 2 \cdot \sqrt[4]{5} )</td>
<td>( 4 \cdot \sqrt[4]{2} )</td>
</tr>
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<td>( = 2 \cdot \sqrt[4]{5} )</td>
<td>( = 4 \cdot \sqrt[4]{2} )</td>
</tr>
<tr>
<td>( = \sqrt[4]{20 \cdot 5} )</td>
<td>( = \sqrt[4]{16 \cdot 2} )</td>
</tr>
<tr>
<td>( = \sqrt[4]{100} )</td>
<td>( = \sqrt[4]{20} )</td>
</tr>
<tr>
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<td>( = \sqrt[4]{20} )</td>
</tr>
<tr>
<td>( = \sqrt[4]{20} )</td>
<td>( = \sqrt[4]{20} )</td>
</tr>
<tr>
<td>( = \sqrt[4]{2048} )</td>
<td>( = \sqrt[4]{2048} )</td>
</tr>
</tbody>
</table>
Ex. 3) Arrange the following in order from greatest to least: \(3\sqrt{5}, 2\sqrt{13}, 4\sqrt{3}, 2, 9\sqrt{2}\)

* Re-write ALL as entire radicals:

\[
\begin{align*}
3\sqrt{5} &= \sqrt{45} \\
2\sqrt{13} &= \sqrt{52} \\
4\sqrt{3} &= \sqrt{48} \\
2 &= \sqrt{4} \\
9\sqrt{2} &= \sqrt{162}
\end{align*}
\]

* now, it is easy to arrange these greatest to least:

\[
\sqrt{162}, \sqrt{52}, \sqrt{48}, \sqrt{45}, \sqrt{4}
\]

* Finally, replace these with the original mixed radicals:

\[
9\sqrt{2}, 2\sqrt{13}, 4\sqrt{3}, 3\sqrt{5}, 2
\]

**Reflection:** How do you use the index of a radical when you write a mixed radical as an entire radical? Use an example to help your explanation.

When you re-write the whole number, you must use the index to determine the new radicand... ex. \(2\sqrt{3}\)

\[
\begin{align*}
2\sqrt{3} &= \sqrt{4 \cdot 3} \\
&= \sqrt{12} \\
&= \sqrt{3^2 \cdot 4} \\
4 &= 2^2 \text{ index}
\end{align*}
\]
4.4 Fractional Exponents and Radicals

Goal: to relate rational exponents and radicals

<table>
<thead>
<tr>
<th>Toolkit:</th>
<th>Main Ideas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Exponent Laws</td>
<td></td>
</tr>
<tr>
<td>• Taking square and cube roots</td>
<td></td>
</tr>
<tr>
<td>• Converting decimals to fractions</td>
<td></td>
</tr>
<tr>
<td>• Order of operations</td>
<td></td>
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Evaluating powers of the form $a^{\frac{1}{n}}$

Powers with Rational Exponents with Numerator 1

When $n$ is as natural number and $x$ is a rational number,

$$x^{\frac{1}{n}} = \sqrt[n]{x} \quad \text{for example...} \quad 16^{\frac{1}{2}} = \sqrt{16} = 4$$

Ex 1) Write each power as a radical then evaluate without using a calculator.

a) $1000^{\frac{1}{3}}$

$$= \sqrt[3]{1000} = \sqrt[3]{10^3} = 10$$

b) $0.25^{\frac{1}{2}}$

$$= \sqrt{0.25} = \sqrt{\frac{1}{4}} = 0.5$$

c) $(-8)^{\frac{1}{3}}$

$$= \sqrt[3]{-8} = -2$$

d) $(16^{\frac{1}{8}})^{\frac{1}{4}} = \frac{16^{\frac{1}{2}}}{8^{\frac{1}{2}}} = \frac{4}{2} = 2$$

Rewriting powers in radical and exponent form

Powers with Rational Exponents

When $m$ and $n$ are natural numbers, and $x$ is a rational number,

$$x^{\frac{m}{n}} = \left(x^\frac{1}{n}\right)^m = \sqrt[n]{x}^m \quad \text{or} \quad (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m} \quad \text{ex} \quad 25^{\frac{1}{3}} = (25^\frac{1}{3})^3 = (\sqrt[3]{25})^3 = (5)^3 = 125$$

Ex 2) Write $26^{\frac{2}{3}}$ in radical form in two different ways.

$\#1 \quad 26^{\frac{2}{3}} = (\sqrt[3]{26})^3$

$\#2 \quad 26^{\frac{2}{3}} = (\sqrt[3]{26})^2 = 5\sqrt[3]{26}$

Ex 3) Write the following in exponent form.

a) $\sqrt[3]{9} \Rightarrow \text{power}$

$$= 9^{\frac{1}{3}}$$

b) $\sqrt[4]{19} \Rightarrow \text{power}$

$$= 19^{\frac{1}{4}}$$

*Think "root" underneath like a tree root. "Power" on top.
### Evaluating powers with rational exponents and rational bases

<table>
<thead>
<tr>
<th>Ex 4) Evaluate the following:</th>
<th>Write as roots!</th>
<th>First change to fraction!</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( 0.123 \rightarrow \sqrt[3]{0.01} )</td>
<td>( (\sqrt[10]{0.1})^3 )</td>
<td>Exponent: ( 1.2 = \frac{12}{10} = \frac{6}{5} )</td>
</tr>
<tr>
<td>b) ((-27)^\frac{4}{3} )</td>
<td>( (3\sqrt[3]{-27})^4 )</td>
<td>( 0.75 )</td>
</tr>
<tr>
<td>c) ( 0.75^\frac{12}{10} )</td>
<td></td>
<td>( (\frac{3}{4})^\frac{6}{5} )</td>
</tr>
</tbody>
</table>

*Use order of operations. Brackets then exponents.

\[
= (0.01)^\frac{3}{10} \\
= 0.001
\]

### Applying rational exponents

Ex 5) Biologists use the formula \( b = 0.01m^{\frac{2}{3}} \) to estimate the brain mass, \( b \) kilograms, of a mammal with body mass, \( m \) kilograms. Use the formula to estimate the brain mass of each animal.

a) A moose with a body mass of 512kg

\[
b = 0.01m^{\frac{2}{3}} \\
\text{We know: } m = 512kg
\]

\[
b = 0.01(512)^{\frac{2}{3}} \quad \text{Change to a radical} \\
b = 0.01(\sqrt[3]{512})^2 \\
b = 0.01(8)^2 \\
b = 0.01(64) \\
b = 0.64 \text{ kg}
\]

The brain mass is 0.64 kg.

b) A cat with a body mass of 5kg

\[
b = 0.01m^{\frac{2}{3}} \\
\text{We know: } m = 5kg
\]

\[
b = 0.01(5)^{\frac{2}{3}} \quad \text{Change to a radical} \\
b = 0.01(\sqrt[3]{5})^2 \\
b = 0.01(1.71)^2 \\
b = 0.01(2.9341) \\
b = 0.03 \text{ kg}
\]

The brain mass is 0.03 kg.

### Reflection

In the power \( x^\frac{m}{n} \), \( m \) and \( n \) are natural numbers and \( x \) is a rational number. What does the numerator \( m \) represent? What does the denominator \( n \) represent? Use an example to explain your answer.

\[
m \rightarrow \text{power} \\
n \rightarrow \text{index of root} \\
x^{\frac{3}{4}} = \sqrt[4]{x^3}
\]
Two numbers with a product of 1 are reciprocals.

Ex1). Since $4 \cdot \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals.

Ex2). Since $\frac{2}{3} \cdot \frac{3}{2} = 1$, the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals.

When $x$ is any non-zero number and $n$ is a rational number, $x^{-n}$ is the reciprocal of $x^n$.

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

Evaluate each power:

Ex. 3): a) $3^{-2}$ \quad b) $(-5)^{-3}$ \quad c) $(-\frac{3}{4})^{-3}$ \quad d) $(\frac{10}{3})^{-2}$

\[
\begin{align*}
\text{a}) & \quad \frac{1}{3^2} = \frac{1}{9} \\
\text{b}) & \quad \frac{1}{(-5)^3} = \frac{1}{-125} \\
\text{c}) & \quad \frac{1}{(-\frac{4}{3})^3} = \frac{27}{64} \\
\text{d}) & \quad \frac{1}{(\frac{3}{10})^2} = \frac{100}{9}
\end{align*}
\]

To evaluate a power with a negative rational (fraction) exponent:

Ex. 4) Evaluate $8^{-\frac{2}{3}}$

\[
\begin{align*}
\text{write with a positive exponent} \\
\text{re-write into radical form, then work from inside out} \\
\text{evaluate (write answer with NO exponents)}
\end{align*}
\]

\[
\begin{align*}
8^{-\frac{2}{3}} & = \frac{1}{8^{\frac{2}{3}}} \\
& = 8^{-2} \\
& = \frac{1}{(2)^2} \\
& = \frac{1}{4} 
\end{align*}
\]
Ex. 5) Evaluate:

a) \((\frac{9}{16})^{\frac{3}{2}}\)  
\[= \left(\frac{3}{4}\right)^{\frac{3}{2}} \quad \text{reciprocate the base, positive exponent}\]
\[= \frac{3^3}{4^3} = \frac{3}{4}\]
\[= \frac{64}{27}\]

b) \((\frac{25}{36})^{\frac{3}{2}}\)  
\[= \left(\frac{25}{36}\right)^{\frac{3}{2}} \quad \text{reciprocate the base, positive exponent}\]
\[= \frac{25}{36}\]
\[= \frac{6}{5}\]

(c) \(16^{\frac{5}{4}}\)  
\[= \left(\frac{1}{16}\right)^{\frac{5}{4}} \quad \text{(Hint: change } \frac{5}{4} \text{ to a fraction in lowest terms)}\]
\[= \frac{1}{16} \quad \text{reciprocate the base, positive exponent}\]
\[= \frac{1}{\sqrt[4]{16}} = \frac{1}{2}\]

(d) \(-25^{-1.5}\)  
\[= -\left(\frac{1}{25}\right)^{\frac{3}{2}} \quad \text{reciprocate the base, negative exponent}\]
\[= \left(-\frac{1}{25}\right)^{\frac{3}{2}} \quad \text{reciprocate the base, positive exponent}\]
\[= \left(-\frac{1}{25}\right)^{\frac{3}{2}} \quad \left(-\frac{1}{25}\right)^{\frac{3}{2}} \quad \text{reciprocate the base, positive exponent}\]
\[= \frac{1}{\sqrt[2]{25}} = \frac{1}{5}\]

Ex. 6) Use the formula \(v = 0.155s^{\frac{3}{2}}f^{-\frac{2}{3}}\) to estimate the speed of a dinosaur when \(s = 1.5\) and \(f = 0.3\) (answer is a speed in \(\text{m/s}\))

Substitute values into the proper places in the formula:

\[v = 0.155 \cdot (1.5)^{\frac{3}{2}} \cdot (0.3)^{-\frac{2}{3}}\]

Evaluate, using your calculator:

\[v = 0.155 \cdot (1.5)^{\frac{3}{2}} \cdot (0.3)^{-\frac{2}{3}}\]
\[v = 0.155 \cdot (1.5)^{\frac{3}{2}} \cdot (0.3)^{-\frac{2}{3}}\]
\[v = 0.155 \cdot (1.9656) \cdot (4.0740)\]

\[v = 1.24 \text{ m/s}\]

The speed of the dinosaur is 1.24 m/s

Reflection:
Yes, I worked all the problems.

Check your work: Yes.
4.6A – Simplifying with Exponent Laws

Goal: to apply all of the exponent laws to simplify expressions

Toolkit:
- Exponent Laws
- Fractional and negative exponents
- Operations with fractions, integers

Main Ideas:

Exponent Laws

Product of powers: \( x^m \cdot x^n = x^{m+n} \)  
(ex: \( x^2 \cdot x^3 = x^{2+3} = x^5 \))

Quotient of powers: \( \frac{x^m}{x^n} = x^{m-n} \)  
(ex: \( \frac{x^y}{x^2} = x^{y-2} = x^2 \))

Power of a power: \((x^m)^n = x^{mn}\)  
(ex: \((x^2)^5 = x^{2 \cdot 5} = x^{10}\))

Power of a product: \((xy)^n = x^ny^n\)  
(ex: \((2x)^2 = 2^2x^2 = 4x^2\))

Power of a quotient: \(\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}\)  
(ex: \(\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}\))

Power of zero: \(x^0 = 1 \forall x \neq 0\) anything b the power zero, except zero, equals 1

Fractional exponents: \(x^{\frac{m}{n}} = \sqrt[n]{x^m} \quad \text{or} \quad (\sqrt[n]{x})^m\)  
(ex: \(x^{\frac{1}{2}} = \sqrt{x^2} \quad \text{or} \quad (\sqrt{x})^2\))

Negative exponents: \(x^{-n} = \frac{1}{x^n}\), \(\frac{1}{x^{-n}} = x^n\), \((\frac{a}{b})^{-m} = (\frac{b}{a})^m\)

Note: write all powers with positive exponents.

Ex 1) Simplify by writing as a single power.

   \(\frac{0.6^2 \cdot 0.6^{-6}}{0.6^4} \quad \text{Ex:} \quad x^{-4} \cdot x^7 \quad m^7 \div m^{-2} \quad \frac{0.4^3}{0.4^4} \quad (n^2)^{-4}\)

   \(0.6^{2-4} = x^{-4+7} = m^{7-(-2)} = 0.4^{3-4} = n^{2-4}\)

   \(0.6^{-4} = x^3 \quad = m^{-2} = 0.4^{-1} = n^{-8}\)

   \(\frac{1}{0.6^4} = \frac{1}{0.4}\)

Which law(s) did you use?
Reflection: How would you simplify the expression \( \frac{\sqrt[3]{x}}{x^2} \) and how is it similar/different compared to the other problems we've done?

Ex. 2) Simplify by writing as a single power:

(a) \( \left( \frac{1}{4} \right)^{-\frac{3}{2}} \)

(b) \( \left( 2x^6 \right)^{-\frac{1}{3}} \)

(c) \( \frac{\sqrt{3}}{\sqrt[3]{2}} \)

Ex. 3) Simplify:

(a) \( \frac{1}{\sqrt[3]{x}} \cdot \frac{1}{\sqrt[3]{y}} \)

(b) \( \frac{1}{x^2} \cdot \frac{1}{y^2} \)

(c) \( \frac{1}{x^3} \cdot \frac{1}{y^3} \)

Note: Write all powers with positive exponents.

Can skip to here.
4.6B – Evaluating with Exponent Laws

**Goal:** to apply all of the exponent laws to evaluate expressions

**Toolkit:**
- Exponent Laws, incl. fractional /negative
- Operations with fractions, integers
- Substitution, BEDMAS

**Main Ideas:**

**What is the difference between “simplifying” and “evaluating”?**

**Simplify:** (write as single base)

Ex 1) Simplify \( x^\frac{5}{2} \cdot x^\frac{1}{2} \)

\[
= x^{\frac{5}{2} + \frac{1}{2}} = x^3
\]

Ex 2) Evaluate \( 1.5^\frac{5}{3} \cdot 1.5^\frac{1}{3} \)

\[
= 1.5^{\frac{5}{3} + \frac{1}{3}} = 1.5^2 = 2.25
\]

Ex 3) Evaluate each expression for \( m = -1 \) and \( n = 2 \)

**Step 1:** Simplify the expression

**Step 2:** Substitute \( \rightarrow \) replace letters with numeric values (\( \text{use brackets!} \))

**Step 3:** Evaluate

\[
\text{a) } (m^2n^3)(m^3n^2) = m^{2+3}n^{3+2} = m^5n^5
\]

\[
\text{b) } (m^{-5}n^{-4})^{-3} = (m^{-5+12}n^{-4-9})^{-3} = m^7n^{13} = m^{-2n}
\]

\[
\text{c) } (m^2n^4)^2 = m^{2\cdot2} = m^4n^8
\]

Ex 4) A sphere has volume 600m\(^3\).

a) Write an expression for the radius in exponent form

\[
V = \frac{4}{3}\pi r^3
\]

\[
3 \times 600 = \frac{4}{3}\pi r^3
\]

\[
r = \left( \frac{1800}{4\pi} \right)^{\frac{1}{3}} = 5.2322
\]

b) What is the radius of the sphere to the nearest tenth of a metre?

\[
r = \left( \frac{1800}{4\pi} \right)^{\frac{1}{3}} = 5.2322
\]

The radius is 5.2m.

**Reflection:** Why is it important to simplify BEFORE evaluating? You can often answer without a calculator, it’s much easier, there are fewer values/operations to deal with.