NB Foundations and Pre-Calculus Math 10 Name: Notes key

Chapter 4 Notes

STUDENT COPY **Final Mark: /8**

Marks → **Requirement ↓**	**2**	**1**	**0**
Notes Present	All notes present	Most notes present	Less than half of notes present
Organization / Neatness	Notes in chronological order, name and date on everything	Almost all notes in chronological order, name and date on most pages	Mostly out of order, name and date often missing
Questions	Question column completed on all notes, higher level questions attempted	Most question columns complete, some higher level questions	Less than half of the question columns complete
Main Ideas and Reflections	All 'main ideas' and 'reflections' complete <u>with care</u> in notes	Most 'main ideas' and 'reflections' complete in notes	Less than half of the 'main ideas' and 'reflections' complete

*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

TEACHER COPY **Final Mark: /8**

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4.1 – Estimating Roots

Name: Notes
Date: Key

Goal: to explore decimal representations of different roots of numbers

Toolkit:	**Main Ideas:**
• Finding a square root • Finding a cube root • Multiplication • Estimating	

Definitions:

Radical: an expression consisting of a radical sign, a radicand, and an index.

index → $\sqrt[n]{x}$ ← radicand } radical

Perfect squares and cubes to memorize: $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{16} = 4$, $\sqrt{25} = 5$, $\sqrt{36} = 6$
$\sqrt{49} = 7$, $\sqrt{64} = 8$, $\sqrt{81} = 9$, $\sqrt[3]{8} = 2$, $\sqrt[3]{27} = 3$, $\sqrt[3]{64} = 4$, $\sqrt[3]{125} = 5$

Ex 1) Evaluate the following radicals, identify the radicand and index for each:

a) $\sqrt{16} = 4$
$4 \times 4 = 16$

Radicand: 16
Index: 2

b) $\sqrt[3]{64} = 4$
$4 \times 4 \times 4 = 64$

Radicand: 64
Index: 3

* if no index is written the index is a 2.

Estimating square roots

Ex 2) Estimate the value of $\sqrt{20}$ to one decimal place.

Step 1: Find the two perfect squares that are closest to the radicand you are looking for (one that is lower and one that is higher).

$\sqrt{20}$
↙ ↘
$\sqrt{16}$ $\sqrt{25}$
↓ ↓
4 5

Step 2: Find which of the two perfect squares is closest to your radicand; this will determine the decimal point of your root.

$20 - 16 = 4$
$25 - 20 = 5$ } $\sqrt{20}$ is closer to $\sqrt{16}$ so the root is closer to 4.

You can check our estimate by quaring it and [illegible] how close you are. → $\boxed{\sqrt{20} \approx 4.4}$

Evaluate $\sqrt{20}$, how close was your estimate?

$\boxed{\sqrt{20} = 4.47}$

Estimating cube roots

Ex 3) Estimate the value of $\sqrt[3]{16}$

Step 1: Find the two perfect cubes that are closest to the radicand you are looking for.

$\sqrt[3]{16}$

$\sqrt[3]{8} \rightarrow 2$ $\qquad$ $\sqrt[3]{27} \rightarrow 3$

Step 2: Find which of the two perfect cubes is closest to your radicand.

$16-8=8 \rightarrow \sqrt[3]{16}$ is closest to $\sqrt[3]{8}$

$27-16=11$

$\boxed{\sqrt[3]{16} \approx 2.4}$

Evaluate $\sqrt[3]{16}$, how close was your estimate?

$\boxed{\sqrt[3]{16} = 2.5}$

Why can you take the cube root of a negative number but not the square root of a negative number?

Ex 4) Estimate the value of $\sqrt[3]{-32}$ $\qquad$ *$(-)(-)(-) = (-)$

$\sqrt[3]{-32}$

$\sqrt[3]{-27} \rightarrow -3$ $\qquad$ $\sqrt[3]{-64} \rightarrow -4$

*$\sqrt{-32}$ is significantly closer to $\sqrt{-27}$, so the root is closer to -3.

$\boxed{\sqrt[3]{-32} \approx -3.2}$

Ex 5) Evaluate $\sqrt{0.64}$

$0.64 = \frac{64}{100}$

$\frac{\sqrt{64}}{\sqrt{100}} = \frac{8}{10} = 0.8$

*If radicand has 2 decimal places, then root has one decimal place

We know $\sqrt{64} = 8$

So, $\boxed{\sqrt{0.64} = 0.8}$

Ex 6) Evaluate $\sqrt{0.0196}$

$0.0196 = \frac{196}{10000}$

$\frac{\sqrt{196}}{\sqrt{10000}} = \frac{14}{100} = 0.14$

*If radicand has 4 decimal places, then root has two.

We know $\sqrt{196} = 14$

So, $\boxed{\sqrt{0.0196} = 0.14}$

Ex 7) Write an equivalent form of 0.3 as a cube root.

$0.3 \times 0.3 \times 0.3 = 0.027 \rightarrow \boxed{\sqrt[3]{0.027}}$

Reflection: How would you write 5 as a square root? A cube root? A fourth root?

Square root $= \sqrt{25} = 5$

Cube root $= \sqrt[3]{125} = 5$

fourth root $= \sqrt[4]{625} = 5$

4.2 – Irrational Numbers

Name: Notes key
Date:

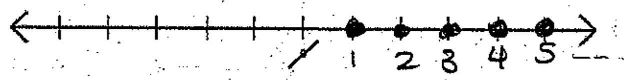
Goal: to classify real numbers, and to identify & order irrational numbers

Toolkit:

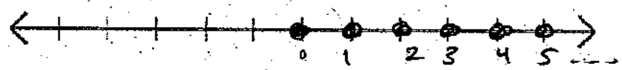
- Estimating roots
- Placing numbers on number lines
- Anything you remember about classifying Real Numbers

Main Ideas:

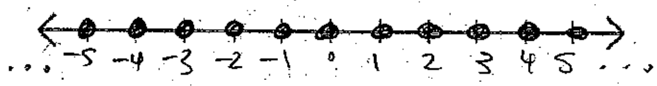
Natural Numbers ($\mathbb{N}$) Counting numbers!



Whole Numbers ($\mathbb{W}$) zero, & counting #s



Integers ($\mathbb{Z}$) negative counting #s, zero, positive counting #s



Rational Numbers ($\mathbb{Q}$) can be written as $\frac{m}{n}$ ← integers ($n \neq 0$)
(fraction!)
Decimals: repeating ($0.\overline{3}$) $\frac{1}{3}$
or terminating (ending - 0.25) $\frac{1}{4}$

Irrational Numbers ($\overline{\mathbb{Q}}$) Not rational! Cannot be written as a fraction;
Decimals do not repeat or terminate. ($\sqrt{2}$, π)

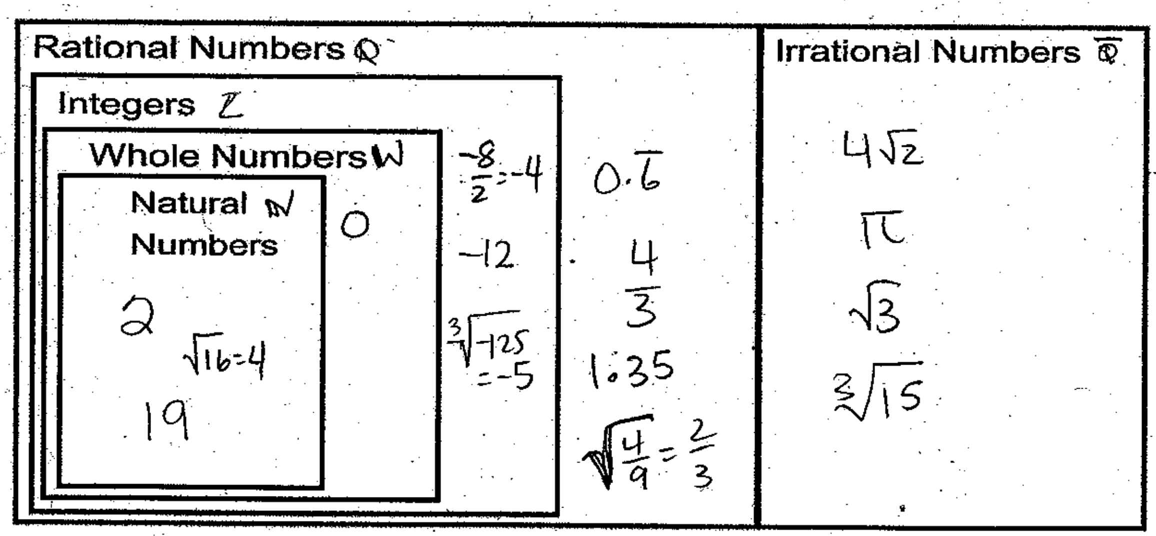
Classifying Real Numbers

Ex1) Where do these numbers belong in the diagram of Real numbers?

$2 \quad 0.\overline{6} = \frac{2}{3}$ (repeats!) $\quad 4\sqrt{2} \quad \frac{4}{3} \quad \frac{-8}{2}$ (−4) $\quad -12 \quad \pi \quad 0 \quad \sqrt{16} = 4$

1.35 (decimal stops "terminates") $\quad \sqrt[3]{-125} = -5 \quad \sqrt{3} \quad \sqrt[3]{15} \quad 19 \quad \sqrt{\frac{4}{9}} = \frac{2}{3}$

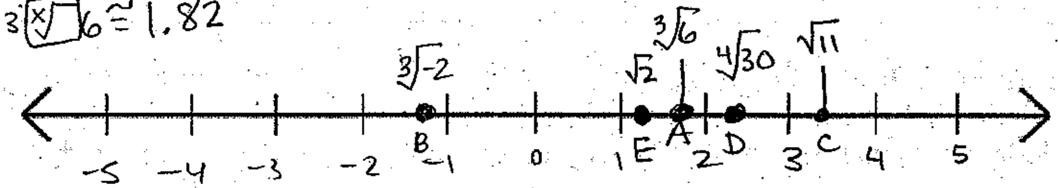
Real Numbers:



Ordering numbers on a number line

Ex2) Use a number line to order these numbers from least to greatest.

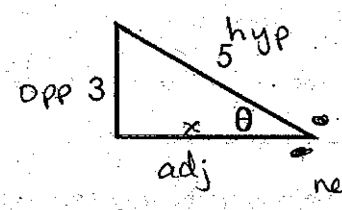
A	B	C	D	E
$\sqrt[3]{6}$	$\sqrt[3]{-2}$	$\sqrt{11}$	$\sqrt[4]{30}$	$\sqrt{2}$
between $\sqrt[3]{1}$ and $\sqrt[3]{8}$, $=1$ $=2$; or use calc: $3\,\boxed{\sqrt[x]{\ }}\,6 \cong 1.82$	Calc: $\cong -1.26$	calc $\cong 3.32$ (or, between $\sqrt{9}$ $=3$ and $\sqrt{16}$ $=4$)	calc $\cong 2.34$	calc $\cong 1.41$



Connect:

Ex3) Is the tangent ratio for θ in each right triangle rational or irrational?

a)



$\boxed{x}$

$x^2+3^2=5^2$

$x^2+9=25$

$-9 \quad -9$

$x^2=16$

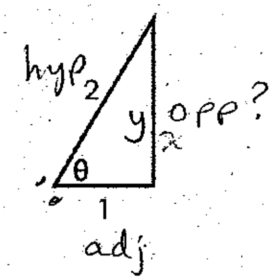
$x=\sqrt{16}$

$x=4$ ← no $\sqrt{\ }$!

$\tan\theta=\frac{opp}{adj}$

$\tan\theta=\frac{3}{4}$ Rational!

b)



Need y!

$\boxed{y}$

$y^2+1^2=2^2$

$y^2+1=4$

$-1 \quad -1$

$y^2=3$

$y=\sqrt{3}$ ← Irrational!

$\tan\theta=\frac{opp}{adj}$

$\tan\theta=\frac{\sqrt{3}}{1}$ ← Irrational!

Reflection: How could you order a set of irrational numbers if you do not have a calculator?

estimate all of the decimals, then place on a number line

4.3A – From Entire to Mixed Radicals

Name: Key
Date:

Goal: to express an entire radical as a mixed radical

Toolkit:
- Understanding Radicals
- Identifying Factors of a Number

Main Ideas:

Perfect Squares - 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ...
1x1 2x2 3x3 4x4 5·5 6·6 7·7 8·8 9·9 10·10 11·11 12·12

Perfect Cubes - 1, 8, 27, 64, 125, 216, ...
1·1·1 2·2·2 3·3·3 4·4·4 5·5·5 6·6·6

What is an entire radical? A radical sign with a number under it. ex. $\sqrt{28}$, $\sqrt[3]{64}$

What is a mixed radical? A number written as the product of a number and a radical. ex. $3\sqrt{5}$, $4\sqrt[3]{10}$

Equivalent Forms:
Ex. 1)

a) $\sqrt{16 \cdot 9}$ is equivalent to $\sqrt{16} \cdot \sqrt{9}$ because:

$\sqrt{144} = 4 \cdot 3$

$12 = 12$ ✓

b) $\sqrt[3]{8 \cdot 27}$ is equivalent to $\sqrt[3]{8} \cdot \sqrt[3]{27}$ because:

$\sqrt[3]{216} = 2 \cdot 3$

$6 = 6$ ✓

What is the Multiplication Property of Radicals?

$\boxed{\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}, \quad \text{where } n \text{ is a natural number, and } a \text{ and } b \text{ are real numbers}}$

***We can use this property to simplify square roots and cube roots that are *not* perfect squares or perfect cubes, but have *factors* that are perfect squares or perfect cubes.**

factors of 24: (1, 2, 3, 4, 6, 8, 12, 24)

Simplifying Square Roots

We can simplify $\sqrt{24}$ because 24 has a perfect square factor of 4.
(hint:look at list of perfect squares!)

- Re-write $\sqrt{24}$ as a product of two factors, with the first one being the perfect square:

$\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ • now simplify $\sqrt{4} = 2$!

$= 2\sqrt{6}$

factors of 24: (1, 2, 3, 4, 6, 8, 12, 24)

Simplifying Cube Roots

We can also simplify $\sqrt[3]{24}$ because 24 has a perfect cube factor of 8.
(hint:look at list of perfect cubes!)

- Re-write $\sqrt[3]{24}$ as a product of two factors, with the first one being the perfect cube:

$\sqrt[3]{24} = \sqrt[3]{8} \cdot \sqrt[3]{3}$ • now simplify $\sqrt[3]{8} = 2$!

$= 2\sqrt[3]{3}$

Ex. 2) Simplify each radical: (remember youflist of perfect squares and perfect cubes!)

Tip: If there is MORE than one perfect square or perfect cube factor, choose the LARGEST one!

a) $\sqrt{80}$

perfect square factors of 80: 4, 16

use 16!

$\sqrt{80} = \sqrt{16} \cdot \sqrt{5}$

$\boxed{= 4\sqrt{5}}$

b) $\sqrt{32}$

perfect square factors of 32: 4, 16

$\sqrt{32} = \sqrt{16} \cdot \sqrt{2}$

$\boxed{= 4\sqrt{2}}$

c) $\sqrt{98}$

perfect square factors of 98: 49

$\sqrt{98} = \sqrt{49} \cdot \sqrt{2}$

$\boxed{= 7\sqrt{2}}$

d) $\sqrt[3]{162}$

perfect cube factor of 162: 27

$\sqrt[3]{162} = \sqrt[3]{27} \cdot \sqrt[3]{6}$

$\boxed{= 3\sqrt[3]{6}}$

e) $\sqrt[3]{108}$

perfect cube factor of 108: 27

$\sqrt[3]{108} = \sqrt[3]{27} \cdot \sqrt[3]{4}$

$\boxed{= 3\sqrt[3]{4}}$

How do you simplify something with an index of 4? (a fourth root?)

Ex. 3) Simplify $\sqrt[4]{162}$

-Rewrite radical with the prime factorization of 162

-Since $\sqrt[4]{162}$ is a fourth root, look for a factor that appears 4 times!

$= \sqrt[4]{81 \cdot 2}$

$= \sqrt[4]{9 \cdot 9 \cdot 2}$

$= \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 2}$ ← prime factorization... and 3 is written 4 times!

$= \sqrt[4]{3^4} \cdot \sqrt[4]{2}$

$\boxed{= 3\sqrt[4]{2}}$

Ex. 4) Simplify $\sqrt[4]{48}$

$= \sqrt[4]{8 \cdot 6}$

$= \sqrt[4]{4 \cdot 2 \cdot 2 \cdot 3}$

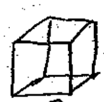
$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$ ← 2 is written 4 times!

$= \sqrt[4]{2^4} \cdot \sqrt[4]{3}$

$\boxed{= 2\sqrt[4]{3}}$

Word Problem

Ex. 5) A cube has a volume of $128cm^3$. Write the edge length of the cube in simplest radical form.



e?

$V = 128cm^3$

$V_{box} = l \cdot w \cdot h$

$V_{cube} = e \cdot e \cdot e$

$V_{cube} = e^3$ → but $V_{cube} = 128cm^3$!

$128 = e^3$ → cube root both sides

so.... $e = \sqrt[3]{128}$ → largest perfect cube factor: ~~8~~, 64

$= \sqrt[3]{64 \cdot 2}$

$= \sqrt[3]{64} \cdot \sqrt[3]{2}$

$e = 4\sqrt[3]{2}cm$

The edge length of the cube is $4\sqrt[3]{2}$ cm

Reflection: How do you use the index of a radical when you simplify a radical? Use an example.

$\sqrt{18}$ → index 2, tells us to look for a perfect square factor

$\sqrt[3]{16}$ → index 3, " " " " " cube factor

general → $\sqrt[n]{a}$ → index n, " " " " power of n factor.

4.3B – From Mixed to Entire Radicals

Name: Key
Date:

Goal: to express a mixed radical as an entire radical

Toolkit:
- List of Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100…
- List of Perfect Cubes: 1, 8, 27, 64, 125, 216, ...
- Multiplication Property of Radicals $(\sqrt{ab} = \sqrt{a} \cdot \sqrt{b})$
- Mixed Radical….ex. $3\sqrt{7}$, $2\sqrt{5}$, $6\sqrt[3]{10}$
- Entire Radical……ex. $\sqrt{105}$, $\sqrt{3}$

Main Ideas:

How do you write a mixed radical as an entire radical?

Write the mixed radical $4\sqrt{3}$ as an entire radical:

$4\sqrt{3}$
$= 4 \cdot \sqrt{3}$ - Use the Multiplication Property of Radicals
$= \sqrt{16} \cdot \sqrt{3}$ (re-write 4 as a radical…..think …..$4 = \sqrt{?}$…..$\sqrt{16}$!)
$= \sqrt{16 \cdot 3}$ - Combine these under the same radical sign and multiply
$= \sqrt{48}$ (***NOTICE…these are the *opposite* steps to writing an entire radical as a mixed radical)

Ex. 1) ~~Write each~~ as an ~~entire radical~~:

a) $5\sqrt{2}$ ($5 = \sqrt{25}$!)
$= 5 \cdot \sqrt{2}$
$= \sqrt{25} \cdot \sqrt{2}$
$= \sqrt{25 \cdot 2}$
$\boxed{= \sqrt{50}}$

b) $3\sqrt{3}$ ($3 = \sqrt{9}$!)
$= 3 \cdot \sqrt{3}$
$= \sqrt{9} \cdot \sqrt{3}$
$= \sqrt{9 \cdot 3}$
$\boxed{= \sqrt{27}}$

c) $3\sqrt[3]{2}$ ($3 = \sqrt[3]{27}$)
$= 3 \cdot \sqrt[3]{2}$
$= \sqrt[3]{27} \cdot \sqrt[3]{2}$
$= \sqrt[3]{27 \cdot 2}$
$\boxed{= \sqrt[3]{54}}$

d) $2\sqrt[3]{6}$ ($2 = \sqrt[3]{8}$)
$= 2 \cdot \sqrt[3]{6}$
$= \sqrt[3]{8} \cdot \sqrt[3]{6}$
$= \sqrt[3]{8 \cdot 6}$
$\boxed{= \sqrt[3]{48}}$

What do you do if the index is 4 or 5 (or higher?)

Write $3\sqrt[5]{2}$ as an entire radical:

First, re-write 3 as $\sqrt[5]{?}$ …. $3 = \sqrt[5]{\underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{\text{5 threes, since index is 5!}}} = \sqrt[5]{243}$

So now, $3\sqrt[5]{2}$
$= 3 \cdot \sqrt[5]{2}$
$= \sqrt[5]{243} \cdot \sqrt[5]{2}$ now, using the Multiplication Property of Radicals…
$= \sqrt[5]{243 \cdot 2}$
$= \sqrt[5]{486}$

Ex. 2) Write each as an entire radical:

a) $2\sqrt[4]{5}$
$= 2 \cdot \sqrt[4]{5}$
$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} \cdot \sqrt[4]{5}$
$= \sqrt[4]{16} \cdot \sqrt[4]{5}$
$= \sqrt[4]{16 \cdot 5}$
$\boxed{= \sqrt[4]{80}}$

b) $4\sqrt[5]{2}$
$= 4 \cdot \sqrt[5]{2}$
$= \sqrt[5]{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} \cdot \sqrt[5]{2}$
$= \sqrt[5]{1024} \cdot \sqrt[5]{2}$
$= \sqrt[5]{1024 \cdot 2}$
$\boxed{= \sqrt[5]{2048}}$

How can entire radicals be used to help you order a set of mixed radicals with the same index?

Ex. 3) Arrange the following in order from greatest to least: $3\sqrt{5}, 2\sqrt{13}, 4\sqrt{3}, 2, 9\sqrt{2}$

* Re-write ALL as entire radicals:

$3\sqrt{5} = \sqrt{9}\sqrt{5} = \boxed{\sqrt{45}}$

$2\sqrt{13} = \sqrt{4}\sqrt{13} = \boxed{\sqrt{52}}$

$4\sqrt{3} = \sqrt{16}\cdot\sqrt{3} = \boxed{\sqrt{48}}$

$2 = \sqrt{4} = \boxed{\sqrt{4}}$

$9\sqrt{2} = \sqrt{81}\sqrt{2} = \boxed{\sqrt{162}}$

* now, it is easy to arrange these greatest to least:

$\sqrt{162}, \sqrt{52}, \sqrt{48}, \sqrt{45}, \sqrt{4}$

* Finally, replace these with the original mixed radicals:

$= \boxed{9\sqrt{2}, 2\sqrt{13}, 4\sqrt{3}, 3\sqrt{5}, 2}$

Reflection: How do you use the index of a radical when you write a mixed radical as an entire radical? Use an example to help your explanation

When you re-write the whole number, you must use the index to determine the new radicand... ex.

$2\sqrt{3} = \sqrt{4}\sqrt{3} = \sqrt{12}$, $4 = 2^{2}$ index

$2\sqrt[3]{3} = \sqrt[3]{8}\sqrt[3]{3} = \sqrt[3]{24}$, $8 = 2^{3}$ index

4.4 Fractional Exponents and Radicals

Name: Notes
Date: Key

Goal: to relate rational exponents and radicals

Toolkit:
- Exponent Laws
- Taking square and cube roots
- Converting decimals to fractions
- Order of operations

Main Ideas:

Evaluating powers of the form $a^{\frac{1}{n}}$

Powers with Rational Exponents with Numerator 1

When n is as natural number and x is a rational number,
$x^{\frac{1}{n}} = \sqrt[n]{x}$ ... for example... $16^{\frac{1}{2}} = \sqrt[2]{16} = 4$

Ex 1) Write each power as a radical then evaluate without using a calculator.

a) $1000^{\frac{1}{3}}$

$\sqrt[3]{1000}$
$= \boxed{10}$

b) $0.25^{0.5}$ Must change to a fraction first!

$0.25^{1/2}$
$\sqrt{0.25}$
$= \boxed{0.5}$

c) $(-8)^{\frac{1}{3}}$

$\sqrt[3]{-8}$
$= \boxed{-2}$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{1/4}}{81^{1/4}}$

$\frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \boxed{\frac{2}{3}}$

Rewriting powers in radical and exponent form

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^{m} = \left(\sqrt[n]{x}\right)^{m}$ ... ex) $25^{\frac{3}{2}} = \left(25^{\frac{1}{2}}\right)^3 = \left(\sqrt[2]{25}\right)^3 = (5)^3 = 125$

$m \rightarrow$ power, $n \rightarrow$ root

or

$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$ ... ex) $25^{\frac{3}{2}} = (25^3)^{\frac{1}{2}} = \sqrt[2]{25^3} = \sqrt{15625} = 125$

Ex 2) Write $26^{\frac{2}{5}}$ in radical form in two different ways.

#1) $26^{2/5} = \left(26^{1/5}\right)^2 = \left(\sqrt[5]{26}\right)^2$

#2) $26^{2/5} = \left(26^{2}\right)^{1/5} = \sqrt[5]{26^2}$

Ex 3) Write the following in exponent form.

*Think "root" underneath like a tree root. "Power" → being on top.

a) $\sqrt{6^5}$ root, → power
$= 6^{5/2}$

b) $\left(\sqrt[4]{19}\right)^3$ root, → power
$= 19^{3/4}$

Evaluating powers with rational exponents and [rat]ional bases	Ex 4) Evaluate the following: Write as roots!

a) $0.01^{\frac{3}{2}}$ → power → root

$\left(\sqrt{0.01}\right)^3$

* Use order of operations. Brackets then exponents

$= (0.1)^3$

$= \boxed{0.001}$

b) $(-27)^{\frac{4}{3}}$

$\left(\sqrt[3]{-27}\right)^4$

$= (-3)^4$

$-3 \cdot -3 \cdot -3 \cdot -3$

$= \boxed{81}$

c) $0.75^{1.2}$ → First change to fraction!

Exponent: $1.2 = \frac{12}{10} = \frac{6}{5}$

$= 0.75^{6/5}$

$= \left(\sqrt[5]{0.75}\right)^6$

$= (0.944)^6$

$= \boxed{0.7080\ldots}$

Applying rational exponents

Ex 5) Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass, m kilograms. Use the formula to estimate the brain mass of each animal.

a) A moose with a body mass of 512kg

$b = 0.01m^{2/3}$ we know: $m = 512$kg

$b = 0.01(512)^{2/3}$ ← change to a radical

$b = 0.01\left(\sqrt[3]{512}\right)^2$

$b = 0.01(8)^2$

$b = 0.01(64)$ The brain mass is 0.64 kg.

$b = 0.64$ kg

b) A cat with a body mass of 5kg

$b = 0.01m^{2/3}$ we know: $m = 5$kg

$b = 0.01(5)^{2/3}$ ← change to a radical

$b = 0.01\left(\sqrt[3]{5}\right)^2$

$b = 0.01(1.71)^2$

$b = 0.01(2.9241)$

$b \doteq 0.03$kg The brain mass is 0.03 kg.

Reflection: In the power $x^{\frac{m}{n}}$, m and n are natural numbers and x is a rational number. What does the numerator m represent? What does the denominator n represent? Use an example to explain your answer.

m → power

n → index of root

$x^{3/4} = \sqrt[4]{x^3}$

4.5 – Negative Exponents and Reciprocals

Name: Key
Date:

Goal: To relate negative exponents to reciprocals

Toolkit:
- Simplifying and evaluating with rational exponents
- Multiplying fractions

Main Ideas:

What is a reciprocal?

Two numbers with a product of 1 are reciprocals.

Ex1). Since $4 \cdot \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals

Ex2). Since $\frac{2}{3} \cdot \frac{3}{2} = 1$, the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals

Powers with Negative Exponents

> When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n.
>
> That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

Evaluate a power with a negative exponent

Evaluate each power:

Ex. 3) a) 3^{-2}

$\frac{1^2}{3^2} = \frac{1}{3^2}$

$\boxed{= \frac{1}{9}}$

b) $(-5)^{-3}$

$= \frac{1}{(-5)^3}$

$= \frac{1}{-125}$

$\boxed{= -\frac{1}{125}}$

c) $(-\frac{3}{4})^{-3}$

$= \left(-\frac{4}{3}\right)^3$

$= \frac{(-4)^3}{3^3}$

$\boxed{= -\frac{64}{27}}$

d) $(\frac{10}{3})^{-2}$

$= \left(\frac{3}{10}\right)^2$

$= \frac{3^2}{10^2}$

$\boxed{= \frac{9}{100}}$

Evaluate a power with a negative *rational* exponent

To evaluate a power with a negative *rational* (fraction) exponent:

Ex. 4) Evaluate $8^{-\frac{2}{3}}$

$= \frac{1}{8^{\frac{2}{3}}}$ power, root — *write with a positive exponent*

$= \frac{1}{(\sqrt[3]{8})^2}$ root, power — *re-write into radical form, then work from inside out*

$= \frac{1}{(2)^2}$ — *evaluate (write answer with NO exponents)*

$= \frac{1}{4}$

Ex. 5) Evaluate:

a) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

$= \left(\frac{16}{9}\right)^{\frac{3}{2}}$ reciprocate the base, positive exponent

$= \frac{16^{\frac{3}{2}}}{9^{\frac{3}{2}}}$

$= \frac{(\sqrt{16})^3}{(\sqrt{9})^3} = \frac{4^3}{3^3}$

$\boxed{= \frac{64}{27}}$

b) $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

$= \left(\frac{36}{25}\right)^{\frac{1}{2}}$

$= \frac{36^{\frac{1}{2}}}{25^{\frac{1}{2}}}$

$= \frac{\sqrt{36}}{\sqrt{25}}$

$\boxed{= \frac{6}{5}}$

c) $16^{-\frac{5}{4}}$

$= \left(\frac{1}{16}\right)^{\frac{5}{4}}$

$= \frac{1^{\frac{5}{4}}}{16^{\frac{5}{4}}}$

$= \frac{1}{(\sqrt[4]{16})^5}$

$= \frac{1}{(2)^5} \boxed{= \frac{1}{32}}$

d) $-25^{-1.5}$

(hint: change 1.5 to a fraction in lowest terms!)

$= -25^{-\frac{3}{2}}$

$= -\left(\frac{1}{25}\right)^{\frac{3}{2}}$

$= -\left(\frac{1}{\sqrt[2]{25}}\right)^3$

$= -\left(\frac{1}{(5)^3}\right)$

$\boxed{= -\frac{1}{125}}$

Applying Negative Exponents (word problems)

Ex. 6) Use the formula $v = 0.155s^{\frac{5}{3}}f^{-\frac{7}{6}}$ to estimate the speed of a dinosaur when $s = 1.5$ and $f = 0.3$ (answer is a speed in m/s)

Substitute values into the proper places in the formula

$$V = 0.155(1.5)^{\frac{5}{3}}(0.3)^{-\frac{7}{6}}$$

brackets

1.5 $\boxed{y^x}\boxed{(}\boxed{5}\boxed{\div}\boxed{3}\boxed{)}\boxed{=}$

0.3 $\boxed{y^x}\boxed{(}\boxed{-}\boxed{7}\boxed{\div}\boxed{6}\boxed{)}\boxed{=}$

Evaluate, using your calculator

$$V = 0.155 \cdot (1.5)^{\frac{5}{3}} \cdot (0.3)^{-\frac{7}{6}}$$

$$V = 0.155 \cdot (1.9656) \cdot (4.0740)$$

$$\boxed{V = 1.24 \text{ m/s}}$$

The speed of the dinosaur is 1.24m/s

Reflection:

Check out the Pattern!

$2^4 = 16$

$2^3 = 8$ $\div 2$

$2^2 = 4$ $\div 2$

$2^1 = 2$ $\div 2$

$2^0 = 1$ $\div 2$

$2^{-1} = \frac{1}{2}$ $\div 2$

$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

$2^{-3} = \frac{1}{8} = \frac{1}{2^3}$

$2^{-4} = \frac{1}{16} = \frac{1}{2^4}$

Would this work for other bases? Yes!

4.6A – Simplifying with Exponent Laws

Name: Notes Key
Date:

Goal: to apply all of the exponent laws to simplify expressions

Toolkit:
- Exponent Laws
- Fractional and negative exponents
- Operations with fractions, integers

Main Ideas:

Exponent Laws

Product of powers: (same base!) $x^m \cdot x^n = x^{m+n}$ ex: $x^2 \cdot x^3 = x^{2+3} = x^5$

Quotient of powers: (same base!) $x^m \div x^n = x^{m-n}$ ex: $\frac{x^4}{x^2} = x^{4-2} = x^2$

Power of a power: $(x^m)^n = x^{m \cdot n}$ ex: $(x^2)^5 = x^{2 \cdot 5} = x^{10}$

Power of a product: $(xy)^m = x^m y^m$ ex: $(2x)^2 = 2^2 \cdot x^2 = 4x^2$

Power of a quotient: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ $y \neq 0$ ex: $\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}$

Power of zero: $n^0 = 1$ $(n \neq 0)$ anything to the power zero, except zero, equals 1.

Fractional exponents: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$ ex: $x^{2/3} = \sqrt[3]{x^2}$ or $(\sqrt[3]{x})^2$

Negative exponents: $x^{-m} = \frac{1}{x^m}$, $\frac{1}{x^{-n}} = x^n$, $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Note: write all powers with POSITIVE EXPONENTS.

Which law(s) did you use?

Ex 1) Simplify by writing as a single power.

same base! same base!

a) $0.6^2 \cdot 0.6^{-6}$ b) $x^{-4} \cdot x^7$ c) $m^7 \div m^{-2}$ d) $\frac{0.4^3}{0.4^4}$ e) $(n^2)^{-4}$

a) $= 0.6^{2+-6}$ $= 0.6^{-4}$ $= \frac{1}{0.6^4}$

b) $= x^{-4+7}$ $= x^3$

c) $= m^{7-(-2)}$ $= m^{7+2}$ $= m^9$

d) $= 0.4^{3-4}$ $= 0.4^{-1}$ $= \frac{1}{0.4}$

e) $= n^{2 \times -4}$ $= n^{-8}$

Power of a Power first! Simplify num/denom separately, then:

Ex 2) Simplify by writing as a single power.

a) $\left[\left(-\frac{4}{7}\right)^{2}\right]^{-3} \div \left[\left(-\frac{4}{7}\right)^{4}\right]^{-5}$

$= \left(-\frac{4}{7}\right)^{-6} \div \left(-\frac{4}{7}\right)^{-20}$

$= \left(-\frac{4}{7}\right)^{-6-(-20)}$

$= \left(-\frac{4}{7}\right)^{-6+20}$

$= \left(-\frac{4}{7}\right)^{14}$

b) $\dfrac{(2.3^{-3})^{-5}}{2.3^{5}}$

$= \dfrac{2.3^{15}}{2.3^{5}}$

$= 2.3^{15-5}$

$= 2.3^{10}$

c) $\dfrac{8^{\frac{5}{4}} \cdot 8^{-\frac{1}{4}}}{8^{\frac{3}{4}}}$

$= \dfrac{8^{\frac{5}{4}-\frac{1}{4}}}{8^{\frac{3}{4}}}$ need common denom. to add/subtr.

$= \dfrac{8^{\frac{4}{4}}}{8^{\frac{3}{4}}}$ leave $\frac{4}{4}$ (common denom!)

$= 8^{\frac{4}{4}-\frac{3}{4}}$

$= 8^{\frac{1}{4}}$

Note: write all powers with POSITIVE EXPONENTS.

Ex 3) Simplify.

all multiplied - reorder!

a) $(x^4y^{-2})(x^2y^3)$

$= x^4 \cdot x^2 \cdot y^{-2} \cdot y^3$

$= x^{4+2}\, y^{-2+3}$

$= x^6 \cdot y^1$

$=$ or x^6y

b) $(27x^6y^9)^{\frac{1}{3}}$ $\quad 9 \times \frac{1}{3} = \frac{9}{3}$

$= 27^{\frac{1}{3}} (x^6)^{\frac{1}{3}} (y^9)^{\frac{1}{3}}$

$= \sqrt[3]{27}\; x^{\frac{6}{3}}\, y^{\frac{9}{3}}$

$= 3x^2y^3$

c) $\left(\dfrac{3\cancel{6}a^4b^{-3}}{7\cancel{14}a^{-2}b^2}\right)^{-2}$ simplify inside, write neg. exp. as pos.

$= \left(\dfrac{3a^4a^2}{7b^3b^2}\right)^{-2}$

$= \left(\dfrac{3a^6}{7b^5}\right)^{-2}$ make + (flip inside)

$= \left(\dfrac{7b^5}{3a^6}\right)^{2}$

$= \dfrac{7^2(b^5)^2}{3^2(a^6)^2}$

$= \dfrac{49b^{10}}{9a^{12}}$

d) $\left(\dfrac{\overset{25}{\cancel{50}}m^2n^4}{\cancel{2}m^4n^2}\right)^{\frac{1}{2}}$ simplify inside

$= \left(25m^{2-4}n^{4-2}\right)^{\frac{1}{2}}$

$= \left(25m^{-2}n^2\right)^{\frac{1}{2}}$

can skip to here

$= \left(\dfrac{25n^2}{m^2}\right)^{\frac{1}{2}}$

$= \dfrac{25^{\frac{1}{2}}n^{\frac{2}{2}}}{m^{\frac{2}{2}}}$

$= \dfrac{5n}{m}$

e) $\dfrac{\left(2x^{\frac{3}{2}}y^2\right)\left(3x^{\frac{1}{2}}y^{-1}\right)}{(4x^3y^{-1})}$ top first

$= \dfrac{{}^{3}\cancel{6}x^{\frac{3}{2}+\frac{1}{2}}y^{2+-1}}{{}^{2}\cancel{4}x^3y^{-1}}$

$= \dfrac{3x^{\frac{4}{2}}y^1}{2x^3y^{-1}}$

$= \dfrac{3x^2y^1}{2x^3y^{-1}}$

$= \dfrac{3x^{2-3}y^{1-(-1)}}{2}$

$= \dfrac{3x^{-1}y^2}{2}$

$= \dfrac{3y^2}{2x}$

Reflection: How would you simplify the expression $\left(\dfrac{x^a}{x^3}\right)^2$ and how is it similar/different compared to the other problems we've done?

$\left(\dfrac{x^a}{x^3}\right)^2 = \dfrac{x^{2a}}{x^6} = x^{2a-6}$ / or $(x^{a-3})^2 = x^{2(a-3)} = x^{2a-6}$

- same laws
- different because we have a variable in the exponent

4.6B – Evaluating with Exponent Laws

Name: Notes Key
Date:

Goal: to apply all of the exponent laws to evaluate expressions

Toolkit:
- Exponent Laws, incl. fractional /negative
- Operations with fractions, integers
- Substitution, BEDMAS

Main Ideas:

* What is the difference between "simplifying" and "evaluating"?

Simplify: (write as single base)

Ex 1) Simplify $x^{\frac{5}{3}} \cdot x^{\frac{1}{3}}$

$= x^{\frac{5}{3}+\frac{1}{3}}$

$= x^{\frac{6}{3}}$

$= x^2$

Evaluate: (write as a single number - no exponents/variables)

Ex 2) Evaluate $1.5^{\frac{5}{3}} \cdot 1.5^{\frac{1}{3}}$

$= 1.5^{\frac{5}{3}+\frac{1}{3}}$

$= 1.5^{\frac{6}{3}}$

$= 1.5^2$

$= 1.5 \times 1.5 = 2.25$

Ex 3) Evaluate each expression for $m = -1$ and $n = 2$

Step 1: Simplify the expression
Step 2: Substitute →replace letters with numeric values (use brackets!)
Step 3: Evaluate

a) $(m^2n^3)(m^3n^2)$

① $m^{2+3}n^{3+2}$

m^5n^5

② $(-1)^5(2)^5$

③ $=(-1)(32)$

$= -32$

b) $\left(\frac{m^{-5}n^5}{m^{-2}n^6}\right)^{-3}$

① $= (m^{-5+2}n^{5-6})^{-3}$

$= (m^{-3}n^{-1})^{-3}$

$= m^9n^3$

② $= (-1)^9(2)^3$

③ $= (-1)(8)$

$= -8$

c) $\frac{(m^n)^2}{m^3}$

① $= \frac{m^{2n}}{m^3}$

$= m^{2n-3}$

② $= (-1)^{2(2)-3}$

$= (-1)^{4-3}$

$= (-1)^1$

$= -1$

Solving Problems using the Exponent Laws

Note: (for H.W.)
$V_{cone} = \frac{1}{3}\pi r^2 h$
if r, h are equal: r=h
$V = \frac{1}{3}\pi h^2 \cdot h$
$V_{cone} = \frac{1}{3}\pi h^3$

Ex 4) A sphere has volume 600m^3.
a) Write an expression for the radius in exponent form
b) What is the radius of the sphere to the nearst tenth of a metre?

a) $V = \frac{4}{3}\pi r^3$

$3 \times 600 = 3 \times \frac{4}{3}\pi r^3$

$\frac{1800}{4\pi} = \frac{\cancel{4\pi} r^3}{\cancel{4\pi}}$

$\left(\frac{1800}{4\pi}\right)^{1/3} = (r^3)^{1/3}$

$r = \left(\frac{1800}{4\pi}\right)^{1/3}$

b) $r \doteq \sqrt[3]{\left(\frac{1800}{4\pi}\right)}$

$r \doteq 5.2322$

The radius is 5.2m.

Reflection: Why is it important to simplify BEFORE evaluating? You can often answer without a calculator; it's much easier; there are fewer values/operations to deal with.