NB *Foundations and Pre-Calculus Math 10* *Name:* Notes Key

Chapter 3 Part 2 Notes

STUDENT COPY **Final Mark: /8**

Marks → Requirement ↓	**2**	**1**	**0**
Notes Present	All notes present	Most notes present	Less than half of notes present
Organization / Neatness	Notes in chronological order, name and date on everything	Almost all notes in chronological order, name and date on most pages	Mostly out of order, name and date often missing
Questions	Question column completed on all notes, higher level questions attempted	Most question columns complete, some higher level questions	Less than half of the question columns complete
Main Ideas and Reflections	All 'main ideas' and 'reflections' complete with care in notes	Most 'main ideas' and 'reflections' complete in notes	Less than half of the 'main ideas' and 'reflections' complete

*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

TEACHER COPY **Final Mark: /8**

Marks → Requirement ↓	**2**	**1**	**0**
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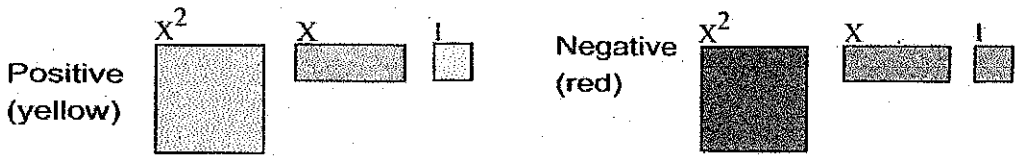
3.7 – ALGEBRA TILES

Name: Notes Key
Date:

Goal: to model polynomial products using algebra tiles

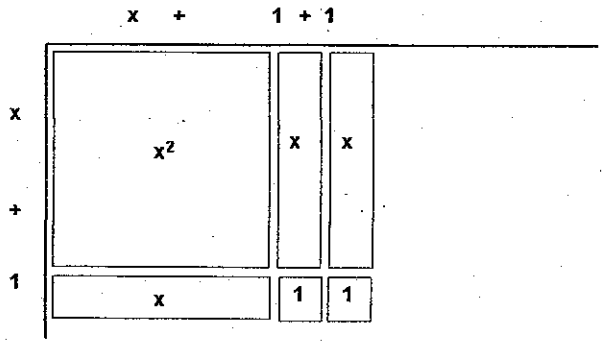
How do we use algebra tiles?

Algebra tiles: (FLIP OVER FOR NEW COLOUR!)



Model:
$(x + 1)(x + 2)$

Make the LENGTH of the rectangle x + 2 and the WIDTH of the rectangle x + 1. Fill in the rectangle, and see how many pieces (and what kinds) you need to do so.

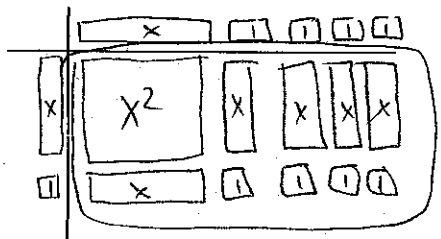


Total Area is $x^2 + 3x + 2$

Note: $(+)(+) = +$
$(+)(-) = -$
$(-)(+) = -$
$(-)(-) = +$

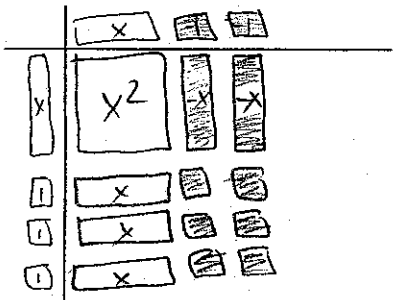
Ex3) Model the following products from using algebra tiles: (*Sketch*!)

a) $(x + 4)(x + 1)$



$x^2 + 5x + 4$

b) $(x + 3)(x - 2)$



$x^2 + 3x - 2x - 6$

$x^2 + x - 6$

Reflection: How (if at all) do the algebra tiles help you picture the multiplication?

3.3 – Common Factors of a Polynomial

Name: Notes
Date: Key

Goal: to determine the factors of a polynomial by identifying the GCF

Toolkit:	**Main Ideas:**
• Finding the GCF • Distributive Property	

Factor a binomial using the GCF

Ex 1) Factor the binomial $3g + 6$

What is the GCF of $3g$ and 6? GCF = 3

$3(g + 2)$ (So factor 3 out of each term)

Check using distributive: $3(g+2) = 3g + 6$ ✓

Ex 2) Factor the binomial $-8y + 16y^2$

GCF of $-8y$ and $16y^2$? GCF = $8y$

$8y(-1 + 2y)$

Check: $8y(-1+2y) = -8y + 16y^2$ ✓

Factor a trinomial using the GCF

Ex 3) Factor the trinomial $3x^2 + 12x - 6$

GCF of $3x^2$, $12x$ and -6? GCF = 3

$3(x^2 + 4x - 2)$

Check: $3(x^2+4x-2) = 3x^2+12x-6$ ✓

Ex 4) Factor the trinomial $6 - 12z + 18z^2$

GCF = 6

$6(1 - 2z + 3z^2)$

Check: $6 - 12z + 18z^2$ ✓

Factor polynomials in more than one variable

Ex 5) Factor the trinomial $-20c^4d - 30c^3d^2 - 25cd$

GCF of $-20c^4d$, $-30c^3d^2$, $-25cd$? GCF = $5cd$

$5cd(-4c^3d - 6c^2d - 5)$

Check: $-20c^4d^2 - 30c^3d^2 - 25cd$ ✓

Could we have factored out a "$-5cd$"? → We usually want the first term to be positive

$-5cd(4c^3d + 6c^2d + 5cd)$ → Check and see if this is also correct?

Reflection: How are the processes of factoring and expanding related?

They are inverses of each other

3.5 – Factoring Trinomials of the form $x^2 + bx + c$, where a=1

Name: Notes Key
Date:

Goal: to use models and algebraic strategies to multiply binomials and to factor trinomials.

Toolkit:
- Factoring

ax^2+bx+c
(# with x^2 | # with x | constant)

Main Ideas:

Definitions:
Descending order: the terms are written in order from the term with the greatest exponent to the term with the least exponent
Ascending order: the terms are written in order from the term with the least exponent to the term with the greatest exponent

Steps for Factoring a Trinomial in the form: x^2+bx+c, where a=1

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable *(biggest to smallest)*
Step 2: Find two numbers that multiply to equal the *c* term and add to equal the *b* term (add to the middle, multiply to the end)
Step 3: Factor into two binomials using the numbers from step 2, with the variable from the question placed first in each bracket

Multiplying two binomials

Ex 1) Expand and Simplify: $(x-1)(x-7)$ use FOIL

$x^2 - 7x - 1x + 7$

$x^2 - 8x + 7$ ($a=1$, $b=-8$, $c=7$)

b=-8 came from adding -7x and -1x.
c=+7 came from multiplying (-1)(-7).

Remember: expanding and factoring are opposite operations... they UNDO each other!

Factoring a trinomial in the form $x^2 + bx + c$

Ex 2) Factor the trinomial: $x^2 - 8x + 7$ ....we should end up with $(x-1)(x-7)$!

① re-order ✓
② need #s that multiply to c=7 and add to b=-8
$1 \times 7 \rightarrow 1+7=8$
$-1 \times -7 \rightarrow (-1)+(-7)=-8$ ✓
③ $(x-1)(x-7)$

> Notice that *a* (the number in front of the x^2) will always end up being 1 in these questions!

Ex 3) Factor: $a^2 - 2a - 8$
($a=1$, $b=-2$, $c=-8$)

gcf? ✓
① reorder? ✓
② need to * to c=-8 and add to b=-2
$-1 \times 8 \rightarrow -1+8=+7$
$1 \times -8 \rightarrow 1+-8=-7$
$-2 \times 4 \rightarrow -2+4=2$
$2 \times -4 \rightarrow 2+-4=(-2)$ ✓
③ magic #s: 2, -4

$(a+2)(a-4)$

Check: FOIL $a^2-4a+2a-8$
$=a^2-2a-8$ ✓
circle answer!

Factoring a trinomial written in ascending order

Ex 4) Factor: $-30 + 7m + m^2$

GCF? ✓
① reorder? $m^2+7m-30$
②
($a=1$, $b=7$, $c=-30$)

* to -30 + to 7
$+3 \times -10 \rightarrow -7$ ✗
$-3 \times +10 \rightarrow +7$ ✓

$(m-3)(m+10)$

Check: $m^2+10m-3m-30$
$m^2+7m-30$ ✓
now circle answer!

Ex 5) Factor: $-5h^2 - 20h + 60$ (÷ -5 ÷ -5 ÷ -5)

Always check to see if there is a GCF you can factor out first! IF there is a negative number in front of the x^2, factor out the negative as well.

$a \neq 1$!

gcf? (-5) $-5(h^2 + 4h - 12)$

copy (-5) and proceed!

$-5(h-2)(h+6)$

① reorder? ✓

② ⊛ to -12 ⊕ to +4

-3, 4 → +1; 3, -4 → -1; -2, +6 → +4; 2, -6 → -4

Check: FOIL first!

$-5(h^2 + 6h - 2h - 12)$

$-5(h^2 + 4h - 12)$

$-5h^2 - 20h + 60$ ✓ circle answer!

Ex 6) Factor: $-12 - 9g + 3g^2$

reorder/gcf: $3g^2 - 9g - 12$

$3(g^2 - 3g - 4)$

$3(g+1)(g-4)$

⊛ to -4 ⊕ to -3

-1, 4 → 3; 1, -4 → -3; 2, -2 → 0

Check: FOIL first!

$3(g^2 - 4g + 1g - 4)$

$3(g^2 - 3g - 4)$

$3g^2 - 9g - 12$ ✓ circle answer

Ex 7) Factor: $2x^2 - 6x - 80$

reorder? gcf?

$2(x^2 - 3x - 40)$

need ⊛ to -40 ⊕ to -3

$2(x-8)(x+5)$

Is -1 × 40 smart to check? not really!

-1 + 40 = 39!

try: (-8 × 5) → -3; 8 × -5 → +3

Check: FOIL first!

$2(x^2 + 5x - 8x - 40)$

$2(x^2 - 3x - 40)$

$2x^2 - 6x - 80$ ✓

Ex 8) Factor: $x^2 + 1x - 2$

⊛ to -2 ⊕ to +1

1 × -2 → -1; -1 × 2 → +1 ✓

$(x-1)(x+2)$

Reflection: Does the order in which the binomial factors are written affect the solution? Explain.

$(x-1)(x+2)$

$= x^2 + 2x - 1x - 2$

$= x^2 + x - 2$

$(x+2)(x-1)$

$= x^2 - 1x + 2x - 2$

$= x^2 + x - 2$

No! You get same product regardless of the order you multiply in.

3.6 – Polynomials of the Form $ax^2 + bx + c$, $a \neq 1$

Name: Notes key
Date:

Goal: to extend the strategies for multiplying binomials and factoring trinomials

Toolkit:	**Main Ideas:**
• Multiplying binomials • Factoring	

Factoring by Decomposition: (needed when the a ≠1 in $ax^2 + bx + c$)

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable *(biggest to smallest)*

Step 2: Find two numbers that multiply to equal *ac* and add to equal *b* *(add to the middle, multiply to **product** of first and last)*

Step 3: Re-write the expression but split or *decompose* the *b* term using the two numbers from step 2.

Step 4: Now the expression has FOUR terms, so we can factor by grouping the first two terms and the last two terms.

Step 5: When fully factored, the remaining two brackets need to be identical! These are now a common factor, and can be factored out, and what is left becomes the components of the second bracket.

Factor by Grouping

take out common "bracket"

Ex. 1) Factor the following by grouping: no gcf for all 4, but each pair has a gcf!

a) $3x^2 - 3x - 2x + 2$

gcf: $3x$ $3x(x-1) - 2(x-1)$ gcf: -2 so match

$(x-1)(3x-2)$

Can check!

b) $2x^2 - 4x + x - 2$

$2x(x-2) + 1(x-2)$

$(x-2)(2x+1)$

Factoring a trinomial of the form $ax^2 + bx + c$

notice that *a* (the number in front of x^2) is not = 1 in any of these questions!

Ex 2) Factor the trinomial: $4g^2 + 11g + 6$ by decomposition

$ac = 4 \times 6 = 24$

$a=4$ $b=11$ $c=6$

✓ gcf

✓ reorder

② Need to ⊛ to 24 ⊕ to 11

$1 \times 24 \rightarrow 25$

$2 \times 12 \rightarrow 14$

$3 \times 8 \rightarrow 11$! ✓

③ Split ⑥ into $3g$ and $8g$

④ grouping

① $4g^2 + 11g + 6$

② ↙ ↘

③ $4g^2 + 3g + 8g + 6$

④ $g(4g+3) + 2(4g+3)$

⑤ $(4g+3)(g+2)$

check:

$(4g+3)(g+2)$

$4g^2 + 8g + 3g + 6$

$4g^2 + 11g + 6$ ✓

Ex 3) Factor the trinomial: $-7m - 10 + 6m^2$

reorder: $6m^2 - 7m - 10$

$ac = -60$

⊛ to -60 ⊕ to -7

$6 \times -10 \rightarrow -4$

$3 \times -20 \rightarrow -17$

$4 \times -15 \rightarrow -11$

$5 \times -12 \rightarrow -8$ ✓

$6m^2 - 7m - 10$

↙ ↘

$6m^2 + 5m - 12m - 10$

$m(6m+5) - 2(6m+5)$

$(6m+5)(m-2)$

$ac = 8 \times -5 = -40$

Ex 4) Factor: $8p^2 - 18p - 5$

⊛ to −40 ⊕ to −18

$2 \times 20 \rightarrow -18$ ✓

$8p^2 - 18p - 5$

$8p^2 + 2p - 20p - 5$

$2p(4p+1) - 5(4p+1)$

$(4p+1)(2p-5)$

Check:

$8p^2 - 20p + 2p - 5$

$8p^2 - 18p - 5$ ✓

Ex 5) Factor: $6x^2 + 14x - 12$

gcf? reorder!

$2(3x^2 + 7x - 6)$ $3 \times -6 = -18$

⊛ to −18, ⊕ to 7

$-18 \times 1 \rightarrow -17$

$-9 \times 2 \rightarrow -7$

$9 \times -2 \rightarrow +7$ ✓

$2(3x^2 + 9x - 2x - 6)$

$2[3x(x+3) - 2(x+3)]$

$2(x+3)(3x-2)$

Check: FOIL first

$2(3x^2 - 2x + 9x - 6)$

$2(3x^2 + 7x - 6)$

$6x^2 + 14x - 12$ ✓

If you can make a trinomial have a=1 by removing a G.C.F., then you can use "the simple way"!

Ex 6) Factor: $3x^2 + 6x - 9$

$3(x^2 + 2x - 3)$ -3

$3(x-1)(x+3)$

⊛ to −3

$1 - 3 \rightarrow -2$

$-1 \quad 3 \rightarrow +2$ ✓

Ex 7) Find an integer to replace □ so that the trinomial can be factored. How many integers can you find?

$4 \times 9 = 36$

$4x^2 + \square x + 9$

must ⊛ to 36!

1×36	$\pm \rightarrow 37$	$4 \times 9 \pm \rightarrow 13$
-1×-36	$\pm \rightarrow -37$	$-4 \times -9 \pm \rightarrow -13$
2×18	$\pm \rightarrow 20$	$6 \times 6 \rightarrow 12$
-2×-18	$\pm \rightarrow -20$	$-6 \times -6 \rightarrow -12$
3×12	$\pm \rightarrow 15$	
-3×-12	$\pm \rightarrow -15$	

10 diff. answers!

Reflection: Will decomposition work if the *a* value of a trinomial is 1? Do an example to prove this.

Yes!

$1x^2 + 5x + 6$

⊛ to $6 \times 1 = 6$ ⊕ to 5

2×3 $2 + 3$

$x^2 + 2x + 3x + 6$

$x(x+2) + 3(x+2)$

$(x+2)(x+3)$

Notes

3.8 – Factoring Special Polynomials

Name: Key
Date:

Goal: to investigate some special factoring patterns

Toolkit:	**Main Ideas:**
• Finding a square root • Finding GCF • Multiplying Polynomials	

Definitions:

Perfect Square Trinomial: a trinomial of the form $a^2 + 2ab + b^2$; it can be factored as $(a + b)^2$
or of the form $a^2 - 2ab + b^2$; it can factored as $(a - b)^2$
Difference of Squares: a binomial of the form $a^2 - b^2$; it can factored as $(a - b)(a + b)$

Draw an area model of a square with side length a + b to represent a prefect square trinomial.

	a	b
a	a^2	ab
b	ab	b^2

Area = $a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

This is a perfect square trinomial

$a^2 + 2ab + b^2$ factors to $(a+b)^2$

$a^2 - 2ab + b^2 \longrightarrow (a-b)^2$

Factoring a perfect square trinomial

Ex 1) Factor the trinomial: $36x^2 + 12x + 1$ } Any common factors? NO

$36x^2 + 12x + 1$

$36x^2$: perfect square $(6x \cdot 6x)$; 1: perfect square (1×1)

$(6x + 1)(6x + 1) = \boxed{(6x+1)^2}$

Check middle term:

$2ab = 2(6)(1) = 12$ ✓

Factoring trinomials in two variables

Ex 2) Factor the trinomial: $5c^2 - 13cd + 6d^2$ } Any common factors? NO

$5c^2 - 13cd + 6d^2$

$5c \quad -3d$

$1c \quad -2d$

$-3cd - 10cd = -13cd$ ✓

$\boxed{(5c - 3d)(c - 2d)}$

Factoring a Difference of Squares

Ex 3) Factor the binomial: $81m^2 - 49$

difference

perfect square

perf. sq.

$\sqrt{81m^2} = 9m$ $\sqrt{49} = 7$

$(9m-7)(9m+7)$

$a^2 - b^2$

$(a-b)(a+b)$

Ex 4) Factor the binomial: $162v^2 - 2w^4$

$w^4 = (w^2)(w^2)$

gcf: 2

$2(81v^2 - w^4)$

$2(9v - w^2)(9v + w^2)$

can you factor further? No.

Ex 5) Factor the binomial: $\frac{x^2}{25} - \frac{y^2}{4}$

$\left(\frac{x}{5}\right)^2 - \left(\frac{y}{2}\right)^2$

$\left(\frac{x}{5} - \frac{y}{2}\right)\left(\frac{x}{5} + \frac{y}{2}\right)$

Reflection: Does a sum of squares factor? Explain. No.

$36n^2 + 4$

try $(6n+2)(6n+2) = 36n^2 + 12n + 12n + 4$

$36n^2 + 24n + 4$

Need opposite signs so middle terms cancel.

3.9 – Factoring Synthesis

Name:
Date:

FACTORING FLOW CHART

STEP 1 Take out COMMON FACTORS (**GCF**)

STEP 2 Ask: How many terms are there?

TWO

Probably a **difference of squares:**

*You need **subtraction** ("difference") and **squares**

$a^2 - b^2 = (a + b)(a - b)$

Diff of Sqs = Conjugates

Example:

$4x^2 - 9 =$

$(2x)^2 - (3)^2 =$

$(2x + 3)(2x - 3)$

THREE

Factoring **trinomials:**

$ax^2 + bx + c$

Type 1: a = 1

Example:

$x^2 - 3x + 2$

Ask: what ADDS to "b" (here –3) & MULTIPLIES to "c" (here +2)

Answer: –1, –2

Write factors

$(x - 1)(x - 2)$

Type 2: a ≠ 1

Example:

$2x^2 - x - 1$

Ask: what ADDS to "b" (here –1) & MULTIPLIES to "ac" (here 2(–1)= – 2)

Answer: –2, 1

Use these to split the middle term into two separate terms:

$2x^2 - x - 1$

$2x^2 - 2x + 1x - 1$

Factor using grouping: See next column ☺

FOUR

Probably **grouping**:

Example:

$2x^2 - 2x + 1x - 1$

Group the first two terms together, and the last two terms together:

$[2x^2 - 2x] + [1x - 1]$

Factor common factors out of each group:

$2x(x - 1) + 1(x - 1)$

You should have two matching brackets. Factor them out:

$(x - 1)(2x + 1)$

STEP 3 Ask: **FF?** Look inside each factor (bracket) and see if you can **FACTOR FURTHER.**