NB *Foundations and Pre-Calculus Math 10* Name: Notes key

Chapter 3 Part 1 Notes

STUDENT COPY **Final Mark: /8**

Marks → **Requirement ↓**	**2**	**1**	**0**
Notes Present	All notes present	Most notes present	Less than half of notes present
Organization / Neatness	Notes in chronological order, name and date on everything	Almost all notes in chronological order, name and date on most pages	Mostly out of order, name and date often missing
Questions	Question column completed on all notes, higher level questions attempted	Most question columns complete, some higher level questions	Less than half of the question columns complete
Main Ideas and Reflections	All 'main ideas' and 'reflections' complete with care in notes	Most 'main ideas' and 'reflections' complete in notes	Less than half of the 'main ideas' and 'reflections' complete

*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

TEACHER COPY **Final Mark: /8**

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3.1 – Factors and Multiples

Name: Notes Key
Date:

Goal: to determine prime factors, greatest common factors, and least common multiples of whole numbers

Toolkit:

- Division
- Multiplication
- Writing repeated multiplication using powers, e.g. $\underbrace{2\times2\times2\times2\times2\times2}_{\text{6 of them!}}= 2^6$

Main Ideas:

Definitions

Factor – a term which divides evenly into another term

Prime number – when a number has only 2 distinct factors (1 and itself). **Examples:** 2, 3, 5, 7, 11, 13, 17, 19, 23

Composite number – when a number has more than 2 factors. **Examples:** 4, 6, 8, 9 = 3×3, 14 = 7×2

Prime factorization – a term written as a product of prime factors

* *every composite number can be expressed as a product of prime factors* *

Greatest common factor (GCF) – the largest term which will divide evenly into a series of separate terms

Least (or Lowest) common multiple (LCM) – the smallest multiple which is common to series of separate terms

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Prime Factorization

Ex1) Write the prime factorization for each of the **composite** numbers:

a) 3 prime √b) 6 composite √c) 45 comp. d) 47 prime ~~e) 3300~~ → e) 3300

6: 2, 3

$6 = 2\cdot3$

45: 5, 9; 9: 3, 3

smallest to largest

$45 = 3\cdot3\cdot5$

$\boxed{45 = 3^2\cdot5}$

3300: 33, 100; 33: 3, 11; 100: 4, 25; 4: 2, 2; 25: 5, 5

$3300 = 2\cdot2\cdot3\cdot5\cdot5\cdot11$

$\boxed{3300 = 2^2\cdot3\cdot5^2\cdot11}$

Finding the GCF by listing all the factors of each number (the rainbow method)

Ex2) Determine the greatest common factor of 126 and 144

Method 1 – list all the factors and find the largest one in common (test: does it ÷2? ÷3? ÷4?)

(write small!)

Rainbow :)

1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126

1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

Greatest common factor is 18.

Finding the GCF by writing the prime factorization of each number

Method 2

1) write the prime factorization for each number
2) highlight the factors that they have in common
3) multiply all the common factors together go get the GCF

126: 2, 63; 63: 3, 21; 21: 3, 7

144: 12, 12; 12: 3, 4; 4: 2, 2

$126 = 2\cdot3\cdot3\cdot7$

$144 = 2\cdot2\cdot2\cdot2\cdot3\cdot3$

multiply common factors:

$2\cdot3\cdot3$

$= 18$

Finding the LCM by listing the first multiples of each number

Ex3) Find the least common multiple of 28, 42, and 63

Method 1 – list the first few multiples of each number until you find (the first, lowest) one in common

Multiples of 28: 28, 56, 84, 112, 140, 168, 196, 224, 252

Multiples of 42: 42, 84, 126, 168, 210, 252

Multiples of 63: 63, 126, 189, 252

· Lowest common multiple doesn't show up until 252!

Finding the LCM by writing the prime factorization of each number

Method 2

1) write the prime factors of each number
2) highlight the greatest power of each prime in ANY of the lists
3) multiply the greatest powers of each prime together to get the LCM

28 → 2, 14 → 2, 7

42 → 2, 21 → 3, 7

63 → 3, 21 → 3, 7

$28 = 2^2 \cdot 7$ $42 = 2 \cdot 3 \cdot 7$ $63 = 3^2 \cdot 7$

$LCM = 2^2 \cdot 3^2 \cdot 7$
$= 4 \cdot 9 \cdot 7$
$= 252$

check 2: 2^2 is highest power of 2

check 3: 3^2 is highest power of 3

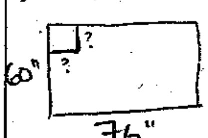
check 7: 7 is highest power of 7

no other prime factors

What types of real-world problems involve GCFs and LCMs?

Ex4) Beside each problem, write whether you would need the GCF or the LCM, then answer the question!

a) A bathroom wall (the part above the bathtub) is a rectangle that measures 78" by 60". If you wanted to cover it exactly with square tiles, what is the largest possible square tile you could use? GCF!



Needs to go evenly into 60 and 76 (smaller – look for FACTORS).

60 → 6, 10 → 2, 3, 2, 5

78 → 2, 39 → 3, 13

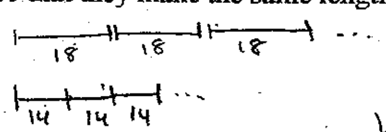
$60 = 2 \cdot 2 \cdot 3 \cdot 5$ $78 = 2 \cdot 3 \cdot 13$

$GCF = 2 \cdot 3$

$GCF = 6$

The largest square tiles you could use are 6" by 6".

b) You have red bungee cords that are 18cm long and green bungee cords that are 14cm long. What is the shortest length of connected bungees you can make with each colour so that they make the same length? LCM!



18 → 2, 9 → 3, 3

14 → 2, 7

$18 = 2 \cdot 3^2$ $14 = 2 \cdot 7$

largest power of 2 anywhere

largest power of 3 anywhere

etc.

$LCM = 2 \cdot 3^2 \cdot 7$

$LCM = 2 \cdot 9 \cdot 7$

$LCM = 126$

The shortest length you could make would be 126cm.

Reflection: How can you remember the difference between a factor and a multiple? Write (or make) a memory trick to help you. (Up to you)

"factors" are pieces "multiples" → multiply (gets bigger)

3.2 – Perfect Squares, Perfect Cubes, and their Roots

Name: Notes key
Date:

Goal: to identify perfect squares and perfect cubes, and to find square roots and cube roots

Toolkit:

- Prime factorization – no calculator!
- The opposite operation of squaring is the square root:

$$5^2 = 25 \ and \ \sqrt{25} = 5$$

- The opposite operation of cubing is the cube root:

$$2^3 = 2 \times 2 \times 2 = 8 \ and \ \sqrt[3]{8} = 2$$

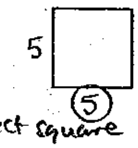
Main Ideas:

What is a Perfect Square?

A **perfect square** is a number that can be written as the product of 2 equal factors.

This means you can represent it as the **AREA OF A SQUARE!** $A = b \times b = b^2$

Picture an actual square! The **square root** is the side length of the square



$A = 5 \times 5 = 25$ Perfect square

square root of 25 is 5

Determining a Square Root

Ex1) Determine the square root of 1296.
Step 1: Write 1296 as a product of its prime factors

1296
2, 648
2, 324
2, 162
2, 81
9, 9
3, 3, 3, 3

$1296 \div 2 = 648$ $\quad$ $324 \div 2 = 162$

$1296 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ $\quad$ Pair up #s

$(2 \cdot 2)(2 \cdot 2)(3 \cdot 3)(3 \cdot 3)$ $\quad$ & split up pairs

Step 2: Re-order the prime factors into TWO identical groups. (If you can't, your number is NOT a perfect square).

$$(2 \cdot 2 \cdot 3 \cdot 3)(2 \cdot 2 \cdot 3 \cdot 3)$$

Step 3: Multiply out each "group" again to see what number it represents $\quad$ $2 \cdot 2 \cdot 3 \cdot 3 = 36$

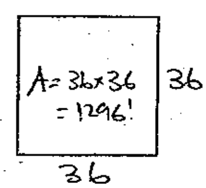
$$(36)(36)$$

* Must be identical! *

Since 1296 can be written as the product of TWO equal factors: 36 × 36, it can be represented as the area of a square.

The square root of 1296 is 36.

We write $\sqrt{1296} = 36$



Terminology: radical, radicand, index:

$$\sqrt[2]{1296}$$

radical

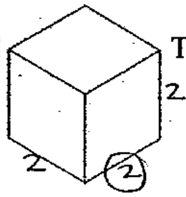
index goes here. "empty" means "2"

radicand: 1296

What is a Perfect Cube?

A **perfect cube** is a number that can be written as the product of 3 equal factors.

This means you can represent it as the **VOLUME OF A CUBE!** $V = e \times e \times e = e^3$

Picture an actual cube! 

The **cube root** is the edge length of the cube.

$V = 2 \times 2 \times 2$

8 perfect cube!

cube root of 8 is 2.

Determining a Cube Root

Ex2) Determine the cube root of 1728. $1+7+2+8=18$ $1+8=9$ divis. by 9!

$9\overline{)1728}$ = 192; 9, 82, 81, 18

Step 1: Write 1728 as a product of its prime factors.

$2\overline{)192}$ = 96; 18, 12

1728 → 9, 192; 9 → 3, 3; 192 → 2, 96; 96 → 3, 32; 32 → 2, 16; 16 → 4, 4; 4 → 2, 2; 4 → 2, 2

$1728 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

$(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(3 \cdot 3 \cdot 3)$ sets of 3!

Step 2: Re-order the prime factors into THREE identical groups. (If you can't, your number is NOT a perfect cube).

$(2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)$

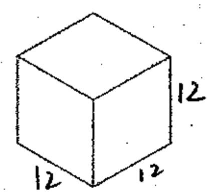
all match! $2 \cdot 2 \cdot 3 = 12$

Step 3: Multiply out each "group" again to see what number it represents

$(12)(12)(12)$

Since 1728 can be written as the product of THREE equal factors: 12 × 12 × 12, it can be represented as the volume of a cube.

The cube root of 1728 is 12.



We write $\sqrt[3]{1728} = 12$

radical, radicand, index?
1728 3

Extend your thinking:

Ex3) Determine the edge length of a cube with volume $64x^6$.

$V = 64x^6$

edge length = cube root

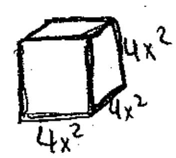
$= \underbrace{2 \cdot 2}\,\underbrace{\cdot 2 \cdot 2}\,\underbrace{\cdot 2 \cdot 2}\cdot \underbrace{x \cdot x}\cdot\underbrace{x \cdot x}\cdot\underbrace{x \cdot x}$

$V = (2 \cdot 2 \cdot x \cdot x)(2 \cdot 2 \cdot x \cdot x)(2 \cdot 2 \cdot x \cdot x)$

$V = (4x^2)(4x^2)(4x^2)$

64 → 8, 8; 8 → 2, 4; 8 → 2, 4; 4 → 2, 2; 4 → 2, 2

edge length $= 4x^2$



Reflection: How could you ESTIMATE the square root or cube root of a number? (Think back to math 9?)

Use square roots of perfect squares.

ex: $\sqrt{5}$ is between $\sqrt{4}$ and $\sqrt{9}$

↓ 2 ↓ 3

2.1? 2.2?

3.7A – Multiplying Monomials & Binomials

Name: Notes key
Date:

Goal: to expand monomial and binomial products (multiply out!)

Toolkit:

- Adding, subtracting, multiplying polynomials
- Multiplying powers with the same base: add the exponents

  Ex: $(x^3)(x^4) = (x \cdot x \cdot x)(x \cdot x x x) = x^7$
- Collecting like terms: same variable(s) with same exponents   Code!

  Ex: $2x^2 + 3x - x^2 + 2x + 1 =$

  $1x^2 + 5x + 1$

Main Ideas:

Definitions

Polynomial – poly = many   nomial = terms (terms seperated by add, subtract)

Monomial – one term   Binomial – 2 terms   Trinomial – 3 terms

e.g. $3x^2y^3$ $(2x^2y + 3xy)$ $(x^2 + 3x + 2)$

Ex1) Expand and simplify → translates to: multiply out and collect like terms

a) $3x^2(x + 3)$

$3x^3 + 9x^2$

think: $(3x^2)(x)$ and $(3x^2)(3)$

$3x^3$ and $9x^2$

b) $(x + 2)(x + 3)$

F O I L

$x^2 + 3x + 2x + 6$

$x^2 + 5x + 6$

stay organized!

First in each bracket : $x \cdot x = x^2$

Outside : $x \cdot 3 = 3x$

Insides : $2 \cdot x = 2x$

Last in each bracket : $2 \cdot 3 = 6$

F O I L

IRST UTSIDE NSIDE AST

…Expand and simplify

c) $(2y+z)(3y-2z)$

$(2y)(3y) + (2y)(-2z) + (z)(3y) + (z)(-2z)$

$6y^2 - 4yz + 3yz - 2z^2$

$6y^2 - yz - 2z^2$

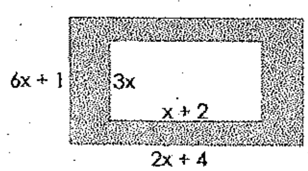
d) $(2a-1)(2a+3)+(a-1)(a-2)$ FOIL twice!

$(4a^2 + 6a - 2a - 3) + (a^2 - 2a - 1a + 2)$ simplify inside each new bracket

$(\boxed{4a^2} + 4a - 3) + (\boxed{a^2} - 3a + 2)$

$5a^2 + a - 1$

Ex2) Find the area of the shaded region (simplified!): $A_{shaded} = A_{big} - A_{little}$ "cut out"



$A_{big\ rect.} = l \times w$

$= (2x+4)(6x+1)$

$= 12x^2 + 2x + 24x + 4$

$= 12x^2 + 26x + 4$

$A_{little\ rect} = l \times w$

$= 3x(x+2)$

$= 3x^2 + 6x$

$A_{shaded} = (12x^2 + 26x + 4) - (3x^2 + 6x)$

$12x^2 \boxed{+26x} + 4 - 3x^2 \boxed{-6x}$

Shaded area: $9x^2 + 20x + 4$

Reflection: Why can you only collect LIKE TERMS? You may wish to use an example to help you explain.

You can't add apples and oranges! $2x + 3x = 5x$

$2 + 5 = 2 + 5$!

$2 + 3 = 5$

3.7B – Multiplying Polynomials

Name: Notes key
Date:

Goal: to expand and simplify polynomials with more than 2 terms

Toolkit:	**Main Ideas:**
• FOIL • Adding, subtracting, multiplying polynomials • Multiplying powers with the same base: add the exponents • Collecting like terms: same variable(s) with same exponents	

Ex1) Expand and simplify:

a) $(x+3y)(x+y-3)$

$x^2 + xy - 3x + 3xy + 3y^2 - 9y$

$x^2 - 3x + 4xy + 3y^2 - 9y$

b) $(x+2)^3$ ← multiply base by itself 3 times!

FOIL one pair

$(x+2)(x+2)(x+2)$

copy

$(x+2)(x^2+2x+2x+4)$ simplify

$(x+2)(x^2+4x+4)$

Note: for questions like $3x^2(x+1)(x+2)$, FOIL first! #19 in homework

line it up cleverly:

$$\begin{array}{rl} x^3 + 4x^2 + 4x & x \text{ times} \ldots \\ + \; 2x^2 + 8x + 8 & + \; 2 \text{ times} \ldots \\ \hline x^3 + 6x^2 + 12x + 8 & \end{array}$$

c) $(r^2+3r-1)(2r^2-r+2)$

$$\begin{array}{ll} r^2 * & 2r^4 - r^3 + 2r^2 \\ 3r * & \quad +6r^3 - 3r^2 + 6r \\ -1 * & \quad\quad -2r^2 + r - 2 \\ \hline & 2r^4 + 5r^3 - 3r^2 + 7r - 2 \end{array}$$