NB

Foundations and Pre-Calculus Math 10 *Name:* Notes keys

Chapter 2 Notes

STUDENT COPY **Final Mark: /8**

Marks → Requirement ↓	**2**	**1**	**0**
Notes Present	All notes present	Most notes present	Less than half of notes present
Organization / Neatness	Notes in chronological order, name and date on everything	Almost all notes in chronological order, name and date on most pages	Mostly out of order, name and date often missing
Questions	Question column completed on all notes, higher level questions attempted	Most question columns complete, some higher level questions	Less than half of the question columns complete
Main Ideas and Reflections	All 'main ideas' and 'reflections' complete <u>with care</u> in notes	Most 'main ideas' and 'reflections' complete in notes	Less than half of the 'main ideas' and 'reflections' complete

*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

TEACHER COPY **Final Mark: /8**

Marks → Requirement ↓	**2**	**1**	**0**
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2.0 – Naming Triangles and Pythagoras

Name: Notes key
Date:

Goal: To learn how to correctly name triangles, their sides and their angles, and to use Pythagoras.

Toolkit:
- Labeling angles and sides of triangles
- All angles in a triangle add to _______
- Pythagoras: $a^2 + b^2 = c^2$ (c is hyp!)
- Labelling triangles from a target angle

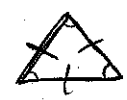
Main Ideas:

Definitions

Right triangle – A triangle with a 90° angle in one corner. (note: other 2 angles must be acute - less than 90°).

right angle

Equilateral triangle – All 3 sides (and 3 angles!) are the same



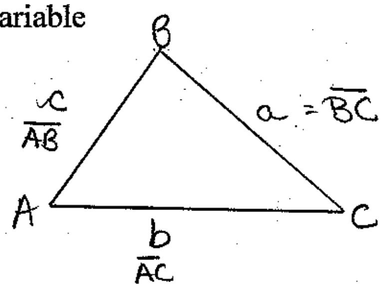
Isosceles triangle – 2 sides (and the 2 angles across from them) are equal.

Scalene triangle – no sides | angles are equal.

Labelling angles and sides of triangles

Ex 1) Draw a triangle, ΔABC, and label all angles and sides.

Label sides using both:
- One lower case variable
- Two endpoints (capital letters!)

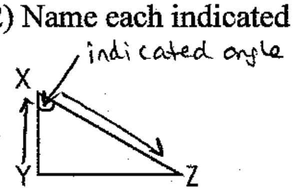


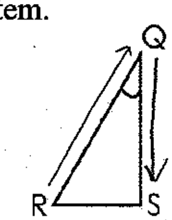
We label the side across from an angle (say, capital A) using the same letter, only in lower case.

Note: Endpoints - capitals.

Three point system of naming angles – An angle is named using the two origins of the angle, and the vertex, with the vertex ALWAYS in the middle!

Ex2) Name each indicated angle using the three point system.





$\angle YXZ$
X is in the middle!
or $\angle ZXY$

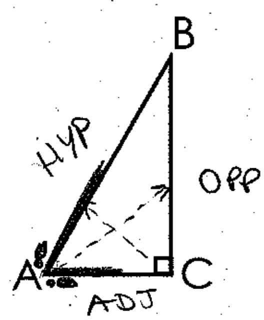
$\angle RQS$
or $\angle SQR$

Labelling angles from a target angle

(Only for right triangles!)

In this chapter, we will also want to label the sides of a RIGHT triangle based their position in relation to a target angle which we use as a reference point.

Ex 3) In reference to angle A, label
-the hypotenuse (HYP)
-the side opposite to A (OPP)
- the side adjacent to A (ADJ)



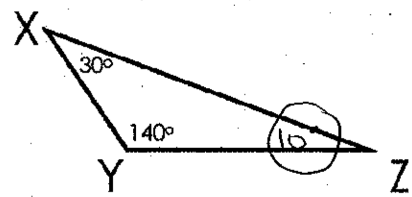
- HYP is always across from 90°
- A is the target angle. from there,
("stand at A" → draw footprints)
label what side is opposite, adjacent (next to, but not the HYP).

Angles in a triangle

The sum of the angles in a triangle is __180°__

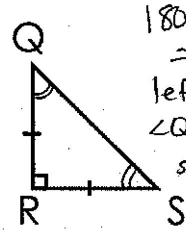
Ex4) Find the missing angle(s).

a)



$\angle Z = 180° - 140° - 30° = 10°$

b)



$180° - 90°$
$= 90°$
left for $\angle Q, \angle S$
$\angle Q = \angle S$ (isosceles!)
so $90° \div 2$
$= 45°$ each

$\angle R = 90°$
$\angle Q = \angle S = 45°$

Pythagoras

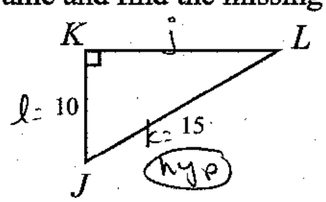
(Only for right triangles!)

Pythagoras – Remember, "c" MUST be the hypotenuse, or the side across from the right angle!

$$a^2 + b^2 = c^2$$

Ex 5) Name and find the missing side(s) (nearest tenth)

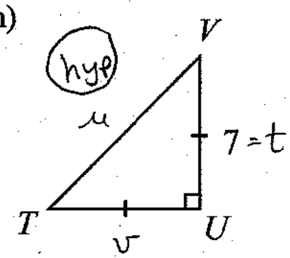
a)



$\boxed{j}$

$j^2 + l^2 = k^2$
$j^2 + 10^2 = 15^2$
$j^2 = 15^2 - 10^2$
$\sqrt{j^2} = \sqrt{125}$
$j = 11.2$

b)



$\boxed{v} = t = 7$

$\boxed{u}$ $v^2 + t^2 = u^2$
$7^2 + 7^2 = u^2$
$\sqrt{98} = \sqrt{u^2}$
$u = 9.9$

Reflection: Is it possible to have an equilateral triangle that is also a right triangle? Explain.

No! Equil. means all angles the same, and you can't have 90°-90°-90°!

2.1 – Angles from the Tangent Ratio

Name: Key.
Date:

Goal: Develop the tangent ratio and relate it to the angle of inclination of a line

Toolkit:	**Main Ideas:**
• Similar Triangles • Labeling sides and angles of a triangle	

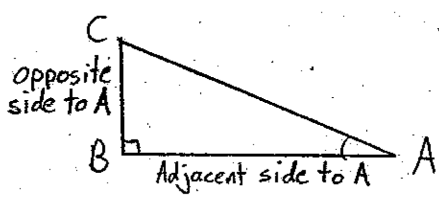
Terminology:

Hypotenuse: The longest side of a right triangle (and always opposite the right angle) (HYP)
Opposite: The side that does NOT touch the angle
(OPP)
Adjacent: The side that DOES touch the angle (and is not the hypotenuse)
(ADJ)

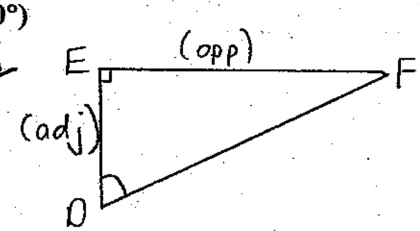
Naming Sides:

We name the sides of a right triangle (a triangle with a 90° angle) in relation to one of its acute angles (one of the angles that is NOT 90°)

Ex. 1)



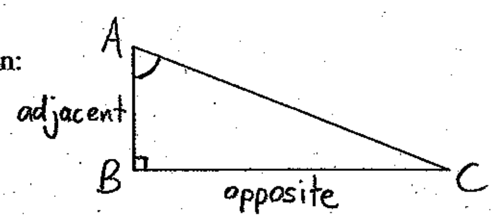
TRY



THE TANGENT RATIO

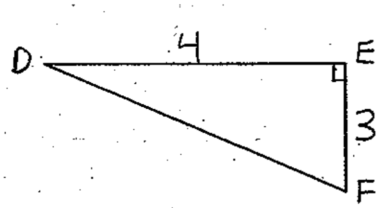
If $< A$ is an acute angle in a right triangle, then:

$$\text{Tan } A = \frac{\textit{length of side opposite} < A}{\textit{length of side adjacent to} < A}$$



Determining the Tangent Ratios for Angles:

Ex. 2) Determine Tan D and Tan F



tan D

$\tan D = \frac{\text{opp to } \angle D}{\text{adj to } \angle D}$

$= \frac{3}{4}$

$\boxed{\tan D = 0.75}$

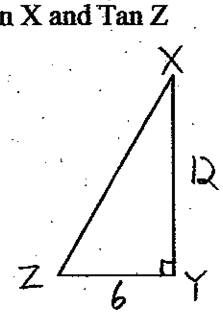
tan F

$\tan F = \frac{\text{opp to } \angle F}{\text{adj to } \angle F}$

$= \frac{4}{3}$

$\boxed{\tan F = 1.\overline{3}}$

Ex. 3) Determine Tan X and Tan Z



tan X

$\tan X = \frac{\text{opp to } \angle X}{\text{adj to } \angle X}$

$= \frac{6}{12}$

$\boxed{\tan X = 0.5}$

tan Z

$\tan Z = \frac{\text{opp to } Z}{\text{adj to } Z}$

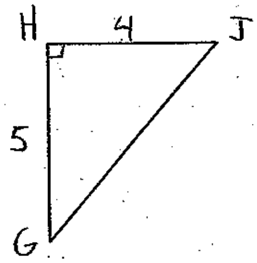
$= \frac{12}{6}$

$\boxed{\tan Z = 2}$

Using the Tan Ratio to Determine the Measure of an Angle:

You can use a scientific calculator to find an angle when you know its tangent. The $\tan^{-1}$ operation does this

Ex. 4) Determine the measures of <G and <J to the nearest tenth of a degree.



$\angle G$

$\tan\angle G = \frac{\text{opp to } \angle G}{\text{adj to } \angle G}$

$\tan\angle G = \frac{4}{5}$

$\tan\angle G = 0.8$

$\angle G = \tan^{-1}(0.8)$

$\boxed{\angle G = 38.7^\circ}$

$\angle J$

$\tan\angle J = \frac{\text{opp to } \angle J}{\text{adj to } \angle J}$

$\tan\angle J = \frac{5}{4}$

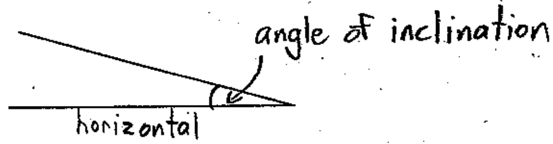
$\tan\angle J = 1.25$

$\angle J = \tan^{-1}(1.25)$

$\boxed{\angle J = 51.3^\circ}$

Definition:

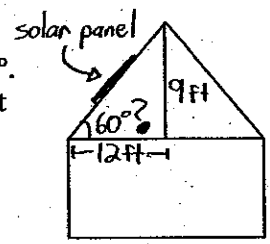
Angle of Inclination – This is the ACUTE angle that a line makes with the horizontal

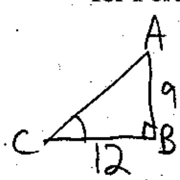


* Make sure your calculator is in "degree" mode!

Using the Tan Ratio to Determine the Angle of Inclination:

Ex. 6) The latitude of Fort Smith, NWT, is approximately 60°. Determine whether this design for a solar panel is best for Fort Smith.





Find $\angle C$

$\tan\angle C = \frac{\text{opp} \angle C}{\text{adj} \angle C}$

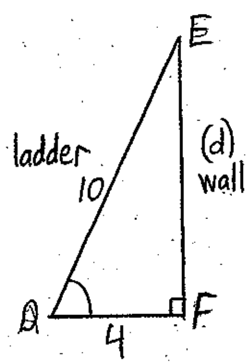
$\tan\angle C = \frac{9}{12}$

$\tan\angle C = 0.75$

$\angle C = \tan^{-1}(0.75) = 37^\circ \rightarrow$ but we need 60° so this is NOT the best design!

Ex. 7) A 10ft ladder leans against the side of a building with its base 4ft from the wall. What angle, to the nearest degree, does the ladder make with the ground?

DRAW A PICTURE!



Find $\angle D$

$\tan D = \frac{\text{opp} \angle D}{\text{adj} \angle D}$

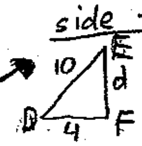
$\tan D = \frac{d}{4}$

$\tan D = \frac{9.17}{4}$

$\tan D = 2.29$

$\boxed{\angle D = \tan^{-1}(2.29) = 66^\circ}$

side trip to find d!



$a^2 + b^2 = c^2$

$4^2 + d^2 = 10^2$

$16 + d^2 = 100$

$\sqrt{d^2} = \sqrt{84}$

$\boxed{d = 9.17}$

The ladder makes an angle of 66° with the ground

Reflection: You have just studied the Tan ratio, which is the ratio of the opposite side to the adjacent side of a right triangle. What are the other two pairs of sides you could have in a right triangle? (think opp, adj, and hyp!) opp - hyp or adj - hyp

2.2 – Sides from the Tangent Ratio Name: Key
Date:

Goal: Apply the tangent ratio to calculate lengths of sides of triangles

Toolkit:
- Similar Triangles
- Labeling sides and angles of a triangle
- Tan Ratio (opposite and adjacent sides)

Main Ideas:

Terminology:

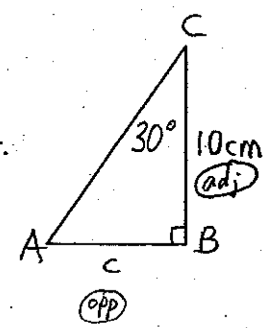
Direct Measurement: When we use a measuring instrument (eg. Ruler, protractor) to determine a length or an angle.

Indirect Measurement: When we use math concepts (eg. Trig, Pythagoras) to calculate a length or an angle

We can use the Tan ratio as a tool to calculate the length of a side of a right triangle *indirectly*.

Steps:
1) Use the Tan ratio ($\frac{opposite}{adjacent}$) to write an equation
2) When we know the measure of an angle (that is NOT the 90° angle!) and the length of one of the legs (not the hypotenuse), solve the equation to determine the length of the other leg.

Determining the Length of a Side **Opposite** a Given Angle:



Ex. 1) Determine the length of AB to the nearest tenth of a centimeter.

*use tangent ratio to write an equation:

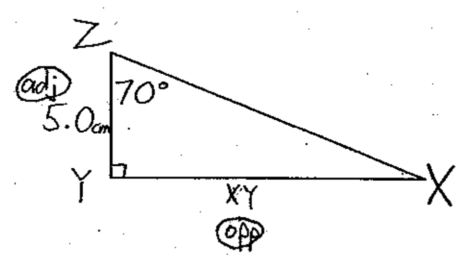
$\tan C = \frac{opp}{adj}$ *we know ∠C = 30°, and adj = 10

$(\tan 30°)^{\times 10} = \left(\frac{c}{10}\right)^{\times 10}$ *solve this for c!

$\boxed{c = 5.8\ cm}$ or $\boxed{AB = 5.8cm}$

Remember Calculator MUST be in DEGREE mode!

Ex. 2) Determine the length of XY to the nearest tenth of a centimeter.



$\tan Z = \frac{opp}{adj}$ *we know ∠Z = 70°, adj = 5.0

$\tan 70° = \frac{XY}{5.0}$ * solve for XY!

$(\tan 70°)^{\times 5.0} = \left(\frac{XY}{5.0}\right)^{\times 5.0}$

$XY = 5.0(\tan 70°)$

$\boxed{XY = 13.7\ cm}$

Remember when solving a question where you have two equal fractions….. *"multiply the pair, divide by the spare!"*

Ex 1. $\frac{\boxed{3}}{1} = \frac{8}{x}$ spare; pair of diagonal #'s you KNOW

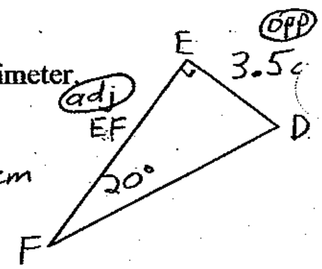
$x = (8 \times 1) \div 3$
$x = 2.67$

Ex 2. $\frac{2.4}{\boxed{1}} = \frac{y}{11}$ pair; spare

$y = (2.4 \times 11) \div 1$
$y = 26.4$

Determining the Length of a Side Adjacent a Given Angle:

Ex. 3) Determine the length of EF to the nearest tenth of a centimeter.



use a tangent ratio!

$\tan F = \frac{opp}{adj}$ *We know ∠F=20°, opp = 3.5cm

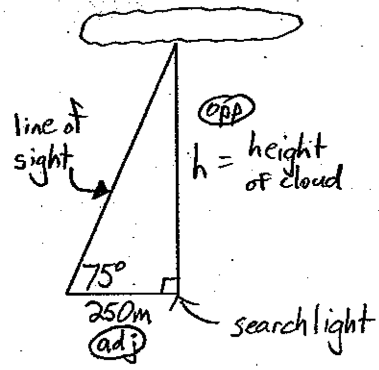
$\tan 20° = \frac{3.5}{EF}$ * "multiply pair, divide by spare"

spare $\frac{\boxed{\tan 20°}}{1} = \frac{3.5}{EF}$ pair $\rightarrow EF = \frac{3.5 \times 1}{\tan 20°} = 9.6$ cm

$\boxed{EF = 9.6 \text{ cm}}$

Using the Tan Ratio to Solve an Indirect Measurement Problem:

Ex. 4) A searchlight beam shines vertically on a cloud. At a horizontal distance of 250m from the searchlight, the angle between the ground and the line of sight to the cloud is 75°. Determine the height of the cloud to the nearest metre.



set up tan ratio!

$\tan 75° = \frac{opp}{adj}$

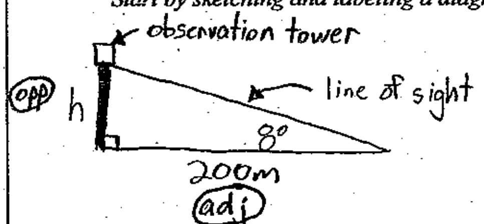
$(\tan 75°)^{\times 250} = \left(\frac{h}{250}\right)^{\times 250}$ solve for h!

$h = 933$

The height of the cloud is 933m

Ex. 5) At a horizontal distance of 200m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is 8°. How high is the observation tower, to the nearest metre?

Start by sketching and labeling a diagram to represent the information in the problem…….



set up tan ratio!

$\tan 8° = \frac{opp}{adj}$

$(\tan 8°)^{\times 200} = \left(\frac{h}{200}\right)^{\times 200}$ solve for h!

$h = 28$m

The observation tower is 28m high

Reflection: Write, in your own words, how you can find the length of a side by using a known angle and a known side, and using the Tan Ratio.

2.4 – The Sine and Cosine Ratios

Name: Notes key
Date:

Goal: to develop and apply the sine and cosine ratios to determine angle measures

Toolkit:

- Labeling sides and angles of a triangle
- What you have learned about the Tan ratio
- Angle of elevation vs depression from horizontal! elev. depr.

(HYP) – **Hypotenuse**: The longest side of a right triangle (and always opposite the right angle)

(OPP) – **Opposite**: The side that does NOT touch the angle

(ADJ) – **Adjacent**: The side that DOES touch the angle (and is not the hypotenuse)

Main Ideas:

THE SINE RATIO

If < A is an acute angle in a right triangle, then

$$\text{Sin } A = \frac{\text{length of side opposite} \angle A}{\text{length of hypotenuse}}$$

THE COSINE RATIO

If < A is an acute angle in a right triangle, then

$$\text{Cos } A = \frac{\text{length of side adjacent} \angle A}{\text{length of hypotenuse}}$$

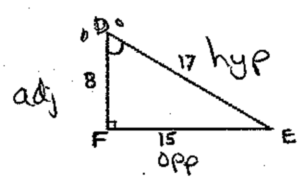
S O H | C A H | T O A

S	O	H	C	A	H	T	O	A
i	p	y	o	d	y	a	p	d
n	p	p	s	j	p	n	p	j

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

Determining the Sine and Cosine of an Angle

Ex1) a) In triangle DEF, identify the side opposite <D, the side adjacent to <D, and the hypotenuse



b) Determine the ratios Sin D and Cos D, and give the values as decimals (nearest hundredth)

$\sin D = \frac{opp}{hyp}$

$\sin D = \frac{15}{17}$

$\sin D = 0.88$

$\cos D = \frac{adj}{hyp}$

$\cos D = \frac{8}{17}$

$\cos D = 0.47$

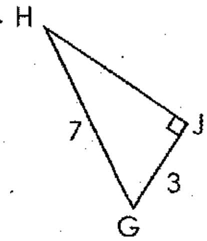
Note: less than one - why?

Using Sine or Cosine to Determine the Measure of an Angle

Remember $\boxed{\cos^{-1}}$ $\boxed{\sin^{-1}}$

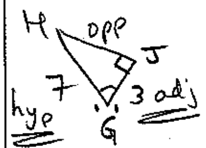
(shift or 2nd F)

Ex2) Determine the measures of <G and <H to the nearest tenth of a degree.



Method 1: start with $\boxed{\angle G}$

① pick an ∠
② label Δ
③ decide sin/cos



have CAH, so use Cos!

$\cos G = \frac{adj}{hyp}$

$\cos G = \frac{3}{7}$

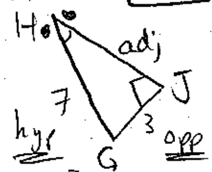
$\cos G = 0.4286$

$\angle G = \cos^{-1}(0.4286)$

$\angle G = 64.6°$

$\angle H = 180° - 90° - 64.6° = 25.4° = \angle H$

Method 2: start with $\boxed{\angle H}$



have SOH, so use Sin!

$\sin H = \frac{opp}{hyp}$

$\sin H = \frac{3}{7}$

$\sin H = 0.4286$

$\angle H = \sin^{-1}(0.4286)$

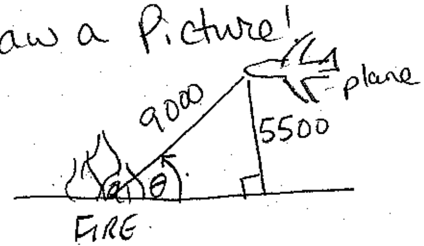
$\angle H = 25.4°$

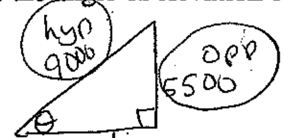
$\angle G = 180° - 90° - 25.4° = 64.6° = \angle G$

Using Sine or Cosine to Solve a Problem

Ex3) A water bomber is flying at an altitude of 5500 ft. The plane's radar shows that it is 9000 ft from the target site in a forest fire. What is the angle of elevation of the plane measured from the target site, to the nearest degree?

Draw a Picture!





angle of elev: with horizontal.

① Pick angle (already chosen)
② label Δ
③ decide sin/cos - you have opp, hyp SOH so SIN!

$\sin \theta = \frac{opp}{hyp}$

$\sin \theta = \frac{5500}{9000}$

$\sin \theta = 0.6111$

$\theta = \sin^{-1}(0.6111)$

$\theta = 37.669$

The angle of elevation is 38°.

Reflection: What will you do to remember the calculator steps when finding an ANGLE (whether it's a sin, cos, or tan problem)?

need angle? shift or 2ndF $\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$

2.5 – Missing Sides from Sine and Cosine

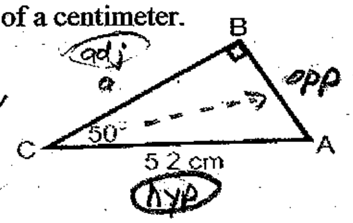
Name: Key
Date:

Goal: Use the sine and cosine ratios to determine lengths indirectly.

Toolkit:	**Main Ideas:**
• What you have learned about the Tan ratio • Angle of elevation vs depression • SOHCAHTOA	

Using the sine or cosine ratio to determine the length of a leg

Ex1) Determine the length of side a to the nearest tenth of a centimeter.



which ratio? we know $\angle C = 50°$, and the hypotenuse is 5.2cm, and side A is ADJACENT $\angle C$, so use ratio that is $\frac{adj}{hyp} \rightarrow$ cos

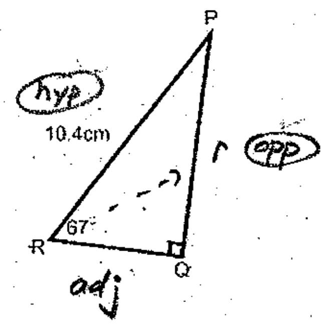
$\cos \angle C = \frac{adj}{hyp}$

$\frac{\cos 50°}{1} = \frac{a}{5.2} \longrightarrow$ $a = 5.2 \times (\cos 50°)$

$\boxed{a = 3.3\text{ cm}}$

Ex2) Determine the length of side r to the nearest tenth of a centimeter.

which ratio is $\frac{opp}{hyp}$? $\rightarrow$ sin!



$\sin \angle R = \frac{opp}{hyp}$

$\sin 67° = \frac{r}{10.4}$

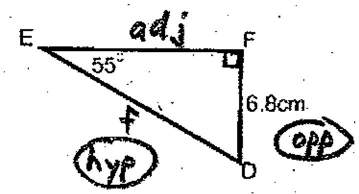
$r = 10.4 \times (\sin 67°)$

$\boxed{r = 9.6\text{ cm}}$

Using the sine or cosine ratio to determine the length of the hypotenuse

Ex. 3) Determine the length of side f to the nearest tenth of a centimeter.

which ratio is $\frac{opp}{hyp}$? $\rightarrow$ sin!



$\sin \angle E = \frac{opp}{hyp}$

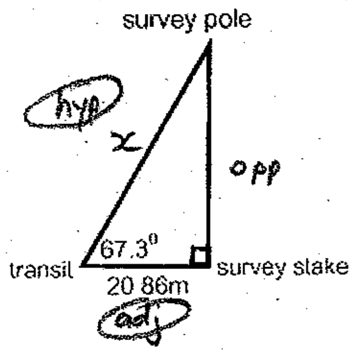
$\frac{\sin 55°}{1} = \frac{6.8}{f}$ * multiply the pair, divide by the spare!

$f = \frac{6.8}{\sin 55°}$

$\boxed{f = 8.3\text{ cm}}$

Solving an Indirect Measurement Problem

Ex. 4) A surveyor made the measurements shown in the diagram. How could the surveyor determine the distance from the transit to the survey pole to the nearest hundredth of a metre?



find x

which ratio?

we have $\frac{adj}{hyp} \rightarrow \cos!$

$$\cos 67.3° = \frac{adj}{hyp}$$

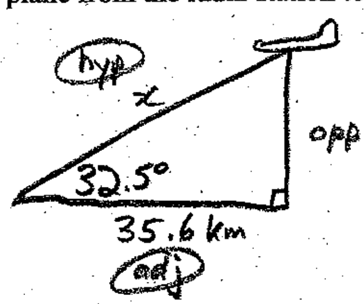
$$\frac{\cos 67.3°}{1} = \frac{20.86}{x}$$

$$x = \frac{20.86}{\cos 67.3}$$

$$\boxed{x = 54.05m}$$

The distance from the transit to the survey pole is 54.05m

Ex. 5) From a radar station, the angle of elevation of an approaching airplane is 32.5°. The horizontal distance between the plane and the radar station is 35.6km. How far is the plane from the radar station to the nearest tenth of a kilometer? (Draw a picture!)



which ratio is $\frac{adj}{hyp}$? $\rightarrow \cos!$

$$\cos 32.5° = \frac{adj}{hyp}$$

$$\frac{\cos 32.5°}{1} = \frac{35.6}{x}$$

$$x = \frac{35.6}{\cos 32.5°}$$

$$\boxed{x = 42.2km}$$

The plane is 42.2km from the radar station.

Reflection: Explain when you might use the sine or cosine ratio instead of the tangent ratio to determine the length of a side in a right triangle.

Any time you need to involve the hypotenuse!

2.6 – Solving Triangles Name: Key

Date:

Goal: Use a trigonometric ratio to solve a problem involving a right triangle

Toolkit:

- $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$ (SOHCAHTOA!)
- The sum of the angles in any triangle is 180°
- Pythagoras → $a^2 + b^2 = c^2$

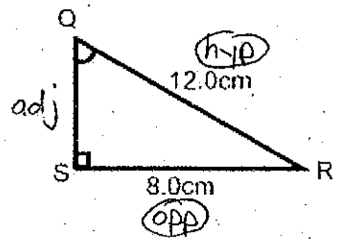
Main Ideas:

Which Trig Ratio should be used?

Find the missing angle or side using trig...

Ex. 1) To determine the measure of the indicated angle or side, which trig ratio would you use? Why? Then find the indicated angle or side, to the nearest tenth of a degree.

a)



*would use $\underline{\sin}$, since we have $\frac{opp}{hyp}$

Find ∠Q: $\sin\angle Q = \frac{opp}{hyp}$

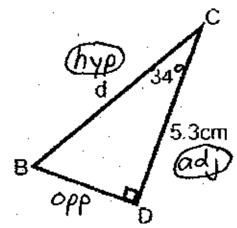
$\sin\angle Q = \frac{8.0}{12.0}$

$\sin\angle Q = (0.6\bar{6})$

$\angle Q = \sin^{-1}(0.6\bar{6})$

$\boxed{\angle Q = 41.8°}$

b)



*would use $\underline{\cos}$, since we have $\frac{adj}{hyp}$

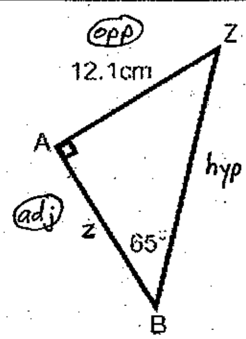
find d: $\cos\angle C = \frac{adj}{hyp}$

$\frac{\cos 34°}{1} = \frac{5.3}{d}$ *multiply pair, divide by spare

$d = \frac{5.3}{\cos 34°}$

$\boxed{d = 6.4\text{cm}}$ * remember units!

c)



*would use $\underline{\tan}$, since we have $\frac{opp}{adj}$

find z:

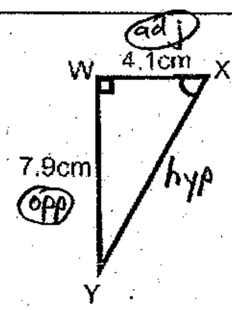
$\tan\angle B = \frac{opp}{adj}$

$\frac{\tan 65°}{1} = \frac{12.1}{z}$

$z = \frac{12.1}{\tan 65}$

$\boxed{z = 5.6\text{m}}$

d)



*would use $\underline{\tan}$, since we have $\frac{opp}{adj}$

find ∠X:

$\tan\angle X = \frac{opp}{adj}$

$\tan\angle X = \frac{7.9}{4.1}$

$\angle X = \tan^{-1}\left(\frac{7.9}{4.1}\right)$

$\angle X = \tan^{-1}(1.9268)$

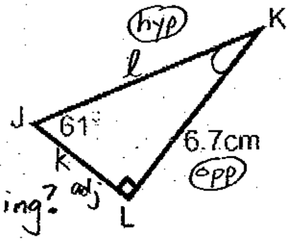
$\boxed{\angle X = 62.6°}$

How do you *SOLVE* a triangle?

Solving a triangle means to determine the measures of all the angles and the lengths of all the sides in a triangle. We will need to use:

- $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$
- The sum of the angles in any triangle is 180°
- Pythagoras $\rightarrow \quad a^2 + b^2 = c^2$

Ex. 2) Solve ΔJKL. Give measures to the nearest tenth.



- "Solve" means find ALL angles and sides.
- what 3 things is this triangle missing?
- ① ∠K ② side l ③ side k
- Find these in ANY order! You may find these using several different tools.

Find ∠K:
3 ∠'s in ANY Δ add to 180°
so... $\angle K = 180° - 90° - \angle J$
$= 180° - 90° - 61°$
$\boxed{\angle K = 29°}$

Find side l:
use sin, we have $\frac{opp}{hyp}$!
$\frac{\sin 61°}{1} = \frac{6.7}{l}$
$l = \frac{6.7}{\sin 61°}$
$\boxed{l = 7.7\text{ cm}}$

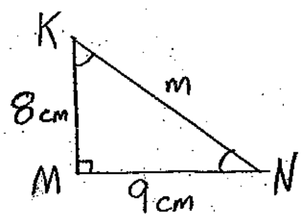
find side k:
*could use trig or pythagoras... You CHOOSE!
use tan, we have $\frac{opp}{adj}$
$\tan 61° = \frac{6.7}{k}$
$k = \frac{6.7}{\tan 61°}$ $\boxed{k = 3.7\text{ cm}}$

$\angle K = 29.0°$, $l = 7.7$cm, k = 3.7cm

How do you solve a triangle without the picture of the triangle?

Ex. 3) In right triangle ΔKMN, $< M = 90°, KM = 8cm$, and $MN = 9cm$. Solve this triangle. Give measures to the nearest tenth.

(Draw and label the triangle, then solve!)



- what 3 things is this triangle missing?
- ① ∠K ② ∠N ③ side m

Find ALL THREE, in any order.

Hint:
* If possible, use numbers from the ORIGINAL question, in case you have made a mistake!

Find ∠K:
use tan, since we have $\frac{opp}{adj}$
$\tan \angle K = \frac{opp}{adj}$
$\tan \angle K = \frac{9}{8}$
$\angle K = \tan^{-1}\left(\frac{9}{8}\right)$
$\boxed{\angle K = 48.4°}$

Find ∠N:
Easy, now that we know ∠K!
$\angle N = 180° - 90° - \angle K$
$= 180° - 90° - 48.4°$
$\boxed{\angle N = 41.6°}$
(could have used tan!)

Find side m:
could use trig or pythag... You CHOOSE!
use pythag.
$a^2 + b^2 = c^2$
$8^2 + 9^2 = m^2$
$64 + 81 = m^2$
$m^2 = 145$
$m = \sqrt{145}$
$\boxed{m = 12.0\text{cm}}$

$\angle K = 48.4°$, $\angle N = 41.6°$, m = 12.0cm

Reflection: What is the advantage of determining the unknown angle before the unknown sides?
Getting angles (180° - 90° - ...) is easy! Do it first!

2.7A – Applications, One Triangle

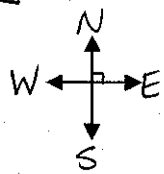
Name: Notes Key
Date:

Goal: to apply trigonometry to solve problems with one right triangle

Toolkit:

- $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$ (SOHCAHTOA!)
- The sum of the angles in any triangle is 180°
- Pythagoras → $a^2 + b^2 = c^2$
- Perimeter – sum of all sides (add!)
- Equilateral, isosceles, scalene

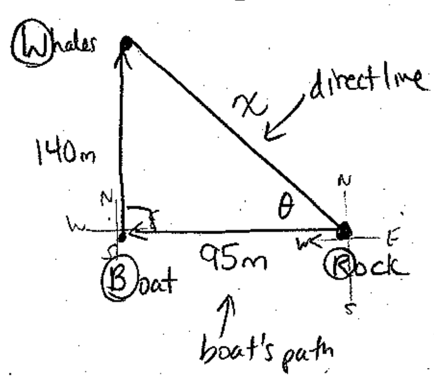
- Compass

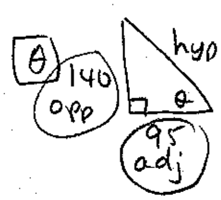


Main Ideas:

Ex1) A whale-watching boat is stopped near a rock to look at some sea lions. Then it goes (95m) due west to head towards a possible whale sighting. The captain points out a pod of whales, which the radar shows are 140m north of the boat. How far are the whales from the sea lions, and what is the angle (at the rock) between the boat's path and the whales' direct line to the sea lions?

hint: start drawing in middle of page so you can go N, S, E or W.





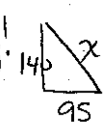
TOA

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{140}{95}$

$\underline{\underline{\theta = 55.8°}}$

$\boxed{x}$ Pythag!



$95^2 + 140^2 = x^2$

$28625 = x^2$

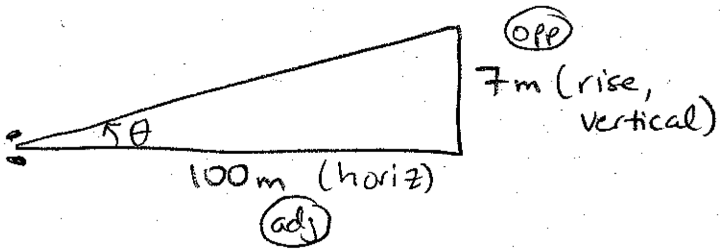
$\underline{\underline{x = 169.2\text{m}}}$

The whales are 169.2m from the sea lions, and the angle at the rock is 55.8°.

Ex2) As Sam is driving, she sees a sign telling her that the road has a 7% grade (i.e., a rise of 7 metres for a horizontal change of 100m).

a) What is the angle of inclination of the road? (nearest degree)
b) If she travels 500m along the road, how much has she risen vertically? (nearest metre)

a) sketch!



TOA

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{7}{100}$

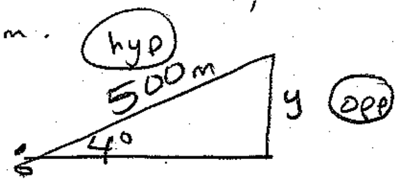
$\tan\theta = 0.07$

$\theta = \tan^{-1}(0.07)$

$\theta = 4.004$

The angle of inclination is 4°.

b) sketch again! This time, new info. Angle = 4°, slanted part is 500m.



SOH!

SOH

$\sin\theta = \frac{opp}{hyp}$

$\sin 4° = \frac{y}{500}$

$y = 500 \sin 4°$

$y = 34.9$

She has risen 35 m vertically.

Reflection: What are two things YOU can do (there are lots!) to help make sure you can solve an application question like the ones in this section?

2.7B – Applications, Two Triangles

Name: Notes Key
Date:

Goal: to apply trigonometry to solve problems with two right triangles

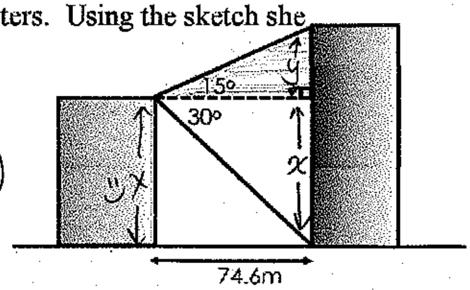
Toolkit:

- $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$ (SOHCAHTOA!)
- The sum of the angles in any triangle is 180°
- Pythagoras → $a^2 + b^2 = c^2$
- A PLAN: you'll need to come up with a PLAN to use more than one triangle to help you answer the question before you jump in.
- Try re-drawing the pieces.

Main Ideas:

2-D

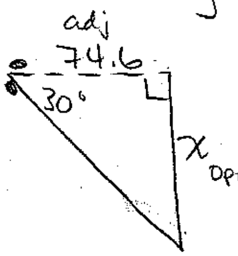
Ex1) From the top of one building, a surveyor measures the angle of elevation to the top of another (taller!) building, and the angle of depression to the base of the other building. The distance between the buildings is 74.6 meters. Using the sketch she made, find the height of the buildings (nearest tenth).

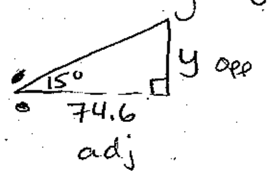


See the 2 triangles?

Plan? Find x (→ small bldg height)

Find y ($x+y$ → tall bldg height)





$\tan\theta = \frac{opp}{adj}$

$\tan 30° = \frac{x}{74.6}$

$x = 74.6 \tan 30°$

$x = 43.1\text{m}$

↑ short bldg

$\tan\theta = \frac{opp}{adj}$

$\tan 15° = \frac{y}{74.6}$

$y = 74.6 \tan 15°$

$y = 19.989$

$y = 20.0\text{m}$

↑

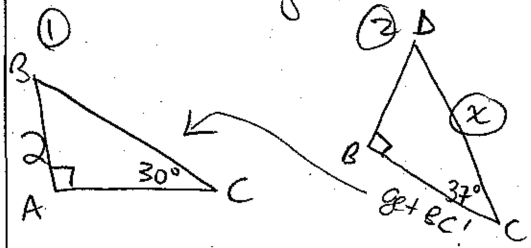
tall bldg $= x + y$
$= 43.1 + 20.0$
$= 63.1\text{m}$

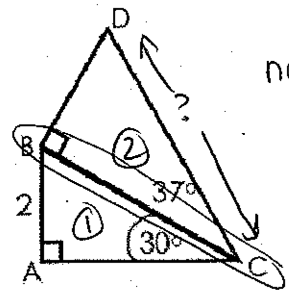
The short building is 43.1m high, and the tall building is 63.1m high.

Ex2) For each questions, write out a PLAN to find the missing side CD.

a) Find the length CD. What's the PLAN?

See the 2 triangles?





not enough info yet. Need help from other triangle!

What do they share? BC.

PLAN:

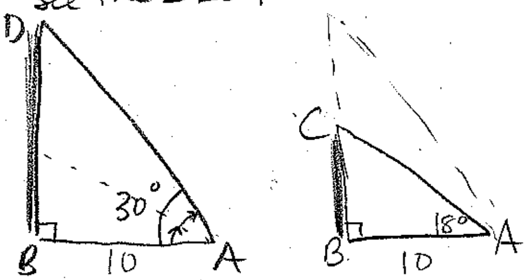
1) Use sin to get BC in △①.

2) Use BC in △②, with cos, to get CD. ☺

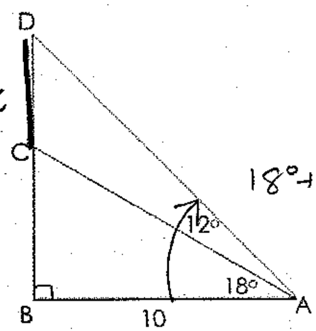
Want to check later? BC = 4
CD = 5.0

b) Find the length CD. What's the PLAN?

See the 2 △s?



? x



18°+12° = 30°

PLAN: Find big height − little height = left over

① Use tan to get BD. (add 18°+12° first!)

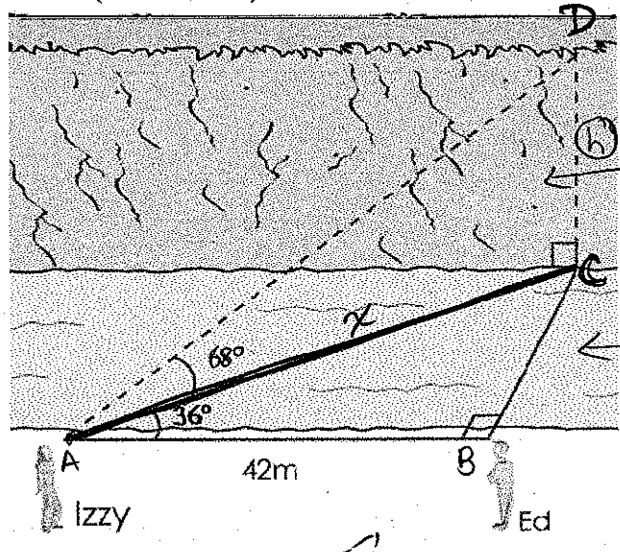
② Use tan to get BC

③ BD − BC = CD ☺

Check later? BD = 5.77
BC = 3.25

CD = 2.52

3-D *Hard to picture: try looking for right angles!*	Ex3) Izzy and Ed positioned themselves 42 m apart on one side of a stream. Izzy recorded angles, as shown below. Find the height of the cliff on the other side of the stream (nearest tenth).



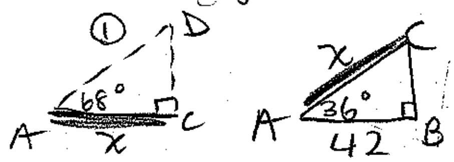
rt Δ "in the air"

right Δ "lying" on stream

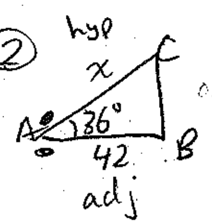
Need (h) in  Not enough info. What does it share? AC!

What's the PLAN?

1. Label Δs, redraw



2. Use ΔABC to get x
3. use x to get height in ΔACD.

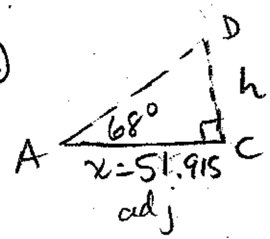
② 

Cos!

$$\cos 36° = \frac{42}{x}$$

$$x = \frac{42}{\cos 36°}$$

$$x = 51.915$$

③ 

tan!

$$\tan 68° = \frac{h}{51.915}$$

$$h = 51.915 \tan 68$$

$$h = 128.5$$

The height of the cliff is 128.5 m.

Reflection: What do you have to think about when you draw a diagram with triangles in three dimensions? Right angles don't look right!